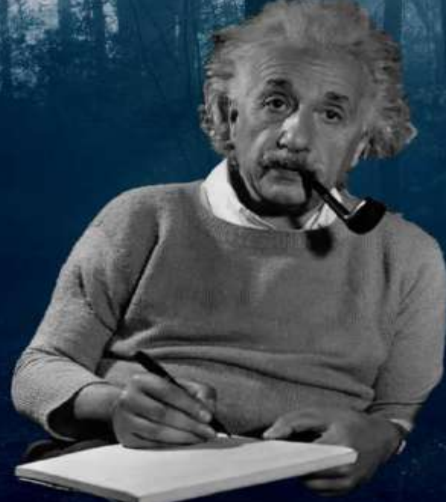
Calculations and Interpretations of the Fundamental constants



BY

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Calculations and Interpretations of The Fundamental Constants

"The only true wisdom is in knowing you know nothing."

— Socrates

By

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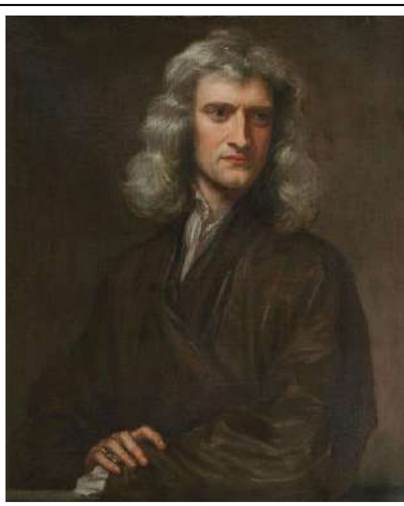
Throughout all of the formulations of the basic equations of gravitation, quantum mechanics, electromagnetism, the nuclear physics and their application to the real world, there appear again and again certain fundamental invariant quantities called the fundamental physical constants – which are generally believed to be both universal in nature and have constant value in time. This book discusses the **calculations and Interpretations of the Fundamental Constants** which consistently appear in the basic equations of theoretical physics upon which the entire scientific study rests, nor are they properties of the **fundamental particles of physics** of which all matter is constituted. The speed of light signifies a maximum speed for any object while the fine-structure constant characterizes the strength of the electromagnetic interaction. An accurate knowledge of fundamental constants is therefore essential if we hope to achieve an accurate quantitative description of our physical universe. The careful study of the numerical values of the **fundamental constants − as determined from various experiments** − can in turn determine the overall consistency and correctness of the basic theories of physics themselves.



I do not know what I may appear to the world, but to myself I seem to have been only like a boy playing on the seashore, and diverting myself in now and then finding a smoother pebble or a prettier shell than ordinary, whilst the great ocean of truth lay all undiscovered before me.

Isaac Newton

A set of **fundamental invariant quantities** that describes the strengths of all the interactions and the physical properties of all the particles observed in nature and appearing in the basic theoretical equations of physics

"It Takes Fundamental Constants To Give Us Our Universe, But They Still Don't Give Everything."

The speed of light (**c**) **(ultimate speed limit)** **186,000 miles per second**	The conversion factor between the time dimension and the three space dimensions in our 4 dimensional space-time If particles with intrinsic mass exceed the speed of light, then c loses its special status, giving rise to a host of other problems elsewhere in the world of physics, where c has been used in calculations, such as the equation in Albert Einstein's theory of special relativity that expresses the equivalence of mass and energy: **$E=mc^2$**
Planck's constant (**h**) {tells about the behavior of the particles and the waves on the atomic scale}	One of the smallest constants used in quantum mechanics that sets the scale for quantum phenomena **($6.626\ 070\ 15 \times 10^{-34}$ J Hz^{-1})**

Planck's constant defines the amount of energy that a electromagnetic radiation photon can carry – according to the frequency of the electromagnetic wave in which it travels

Newtonian gravitational constant (**G**) The basis of our understanding of **non-relativistic gravity**	One of the earliest fundamental constants that defines the strength of gravitational force

The constant relating the force of gravitational attraction between two objects to the product of their masses and the inverse square of the distance between them in Sir Isaac Newton's universal law of gravitation:

$$F = \frac{G m_1 m_2}{r^2}$$

6.673×10^{-11} N m^2 kg^{-2}

The **Boltzmann constant** (k_B) relates temperature to energy. It is a fundamental constant of physics occurring in nearly every statistical formulation of both classical and quantum physics. It is named after Austrian physicist and philosopher **Ludwig Boltzmann**, one of the pioneers of statistical mechanics.

PLANCK FORCE:

The idea of Quantum foam was devised by John Wheeler in 1955

The amount of force required to accelerate one Planck mass by one Planck acceleration:

Planck force = Planck mass × Planck acceleration

$$\frac{c^4}{G} = \sqrt{\frac{\hbar c}{G}} \times \sqrt{\frac{c^7}{\hbar G}}$$

$$\mathbf{F_{Planck} = 1.2103 \times 10^{44}\ N}$$

The maximum force value that can be observed in nature

appears in the Albert Einsteinian field equations describing the properties of a gravitational field surrounding any given mass:

$$\textbf{Einstein tensor} = 8\pi \times \frac{\text{energy–momentum tensor}}{\text{Planck force}}$$

The Planck force describes how much or how easily space-time is curved by a given amount of mass-energy.

The amount of energy possessed by a Schwarzschild Black Hole is equal to its mass multiplied by the square of the speed of light: $E = Mc^2$, where: c is not just the constant namely the maximum distance a light can travel in one second in vacuum but rather a fundamental feature of the way space and time are unified to form space-time.

$$E = \frac{F_{Planck}}{2} \times r_S$$

This means: Half of the Planck force is responsible for confining the energy $E = Mc^2$ of the Black Hole to a distance $r_S = \frac{2GM}{c^2}$.

The value of h is about 0.6 trillionths of a trillionth of a billionth of 1 joule-second.

Any object with a physical radius < $\frac{2GM}{c^2}$ will be a Black Hole.

$\Delta p\ \Delta x \geq \frac{\hbar}{2}$	$\Delta E\ \Delta t \geq \frac{\hbar}{2}$
Planck momentum × Planck length = $\hbar$	Planck energy × Planck time = $\hbar$

The Planck time is the time it takes for light to traverse a Planck length.

The Planck mass is so large because the gravitational force in this universe is very weak

The **Planck mass** is approximately the mass of a black hole where quantum and gravitational effects are at the same scale: where its reduced Compton wavelength and half of its Schwarzschild radius are approximately the same.

If $\sqrt{\frac{\hbar c^5}{G}}$ is confined to the volume of a cube of size $\sqrt{\frac{\hbar G}{c^3}}$ it will form a black hole. In fact, this is thought to be the smallest possible mass limit for a black hole and at

Distance = $\sqrt{\frac{\hbar G}{c^3}}$

Time = $\sqrt{\frac{\hbar G}{c^5}}$

Energy = $\sqrt{\frac{\hbar c^5}{G}}$

it is thought that quantum gravitational effects will be very significant.

at

- Space-time would become chaotic quantum foam. Matter and antimatter would be constantly created and destroyed.
- Space-time would become quantized (which would cause violations of Lorentz invariance).

The attempt to understand the Hawking radiation has a profound impact upon the understanding of the Black Hole thermodynamics, leading to the description of what the black hole entropic energy is:

Black Hole Entropic Energy = Black Hole Temperature × Black Hole Entropy

$$E_S = T_{BH} \times S_{BH} = \frac{Mc^2}{2}$$

This means that the entropic energy makes up half of the mass energy of the Black Hole. For a Black Hole of one solar mass ($M_\odot = 2 \times 10^{30}$ kg), we get an entropic energy of 9×10^{46} joules – much higher than the thermal entropic energy of the sun. Given that power emitted in Hawking radiation is the rate of energy loss of the black hole: $P = -\frac{dMc^2}{dt} = 2 \times -\frac{dE_S}{dt}$. The more power a black hole radiates per second, the more entropic energy being lost in Hawking radiation. However, the entropic energy of the black hole of one solar mass is about 9×10^{46} joules of which only 4.502×10^{-29} joules per second is lost in Hawking radiation.

$$E_S = \frac{F_{Planck}}{4} \times r_S$$

This means: $\frac{1}{4}$ th of the Planck force is responsible for confining the entropic energy $E_S = (T_{BH} \times S_{BH})$ of the Black Hole to a distance $r_S = \frac{2GM}{c^2}$. A photon sphere or photon ring is an area or region of space where gravity is so strong that photons are forced to travel in orbits. The radius of the photon sphere for a Schwarzschild Black Hole: $r = \frac{3GM}{c^2}$**.** This equation entails that photon spheres can only exist in the space surrounding an extremely compact object (a Black Hole or possibly an "**ultracompact**" neutron star).

E = hυ

The first "**quantum**" expression in history – stated by Max Planck in 1900

$$E = \frac{F_{Planck}}{3} \times r$$

This means: $\frac{1}{3}$ rd of the Planck force times the radius of the photon sphere equals the amount of energy possessed by a Schwarzschild Black Hole.

Radiation Constants:

"Nature shows us only the tail of the lion. But there is no doubt in my mind that the lion belongs with it even if he cannot reveal himself to the eye all at once because of his huge dimension. We see him only the way a louse sitting upon him would." **— Albert Einstein**

Fundamental physical constants characterizing black body radiation. The first radiation constant is $c_1 = 2\pi hc^2 = 3.7417749 \times 10^{-16}$ Wm^2, the second is $c_2 = \frac{hc}{k_B} = 1.438769 \times 10^{-2}$ mK, where: h is the Planck constant c is the speed of light in vacuum and k_B the Boltzmann constant.

Radiation

Heat flows through space by means of electromagnetic waves

$$\frac{(\text{elementary charge})^2}{(\text{Planck charge})^2} = \frac{1}{4} Z_0 G_0$$

elementary charge $= \frac{\text{Planck charge}}{2}\sqrt{Z_0 G_0}$

Fine structure constant:

{ **Sommerfeld's constant** }

$$\alpha = \frac{e^2}{4\pi\varepsilon_0 \hbar c} = \frac{1}{4} Z_0 G_0$$

$$\alpha = \frac{1}{4} \times \text{impedance of free space} \times \text{conductance quantum} = 0.0072973525693$$

→ expresses the strength of the electromagnetic interaction between elementary charged particles.

→ $\frac{(\text{elementary charge})^2}{(\text{Planck charge})^2}$

If $\frac{e^2}{4\pi\varepsilon_0 \hbar c}$ were greater than 0.1, stellar fusion would be impossible and no place in the cosmos would be warm enough for carbon-based life as we know it.

When I die my first question to the Devil will be: What is the meaning of the fine structure constant?

— **Wolfgang Pauli**

Planck units

The ultra-high-energy cosmic ray observed in 1991 had a measured energy of about $2.5\times10^{-8}\sqrt{\frac{\hbar c^5}{G}}$

Planck mass	$m_{Planck} = \sqrt{\frac{\hbar c}{G}} = 2.176434 \times 10^{-8}$ kg
Planck length	$L_{Planck} = \sqrt{\frac{\hbar G}{c^3}} = 1.616255 \times 10^{-35}$ m At which all the fundamental forces are unified.
Planck time	$t_{Planck} = \sqrt{\frac{\hbar G}{c^5}} = 5.391247 \times 10^{-44}$ s **Quantum effects of gravity** dominate physical interactions at this time interval.
Planck temperature	$T_{Planck} = \sqrt{\frac{\hbar c^5}{G k_B^2}} = 1.416784 \times 10^{32}$ K At this temperature, the wavelength of emitted **thermal radiation** reaches the **Planck length.**
Planck charge	$q_{Planck} = \sqrt{4\pi\varepsilon_0 \hbar c} = 1.875546 \times 10^{-18}$ C ≈ 11.7e
Planck area	$L_{Planck}^2 = \frac{\hbar G}{c^3} = 2.6121 \times 10^{-70}$ m^2
Planck volume	$L_{Planck}^3 = \sqrt{\frac{\hbar^3 G^3}{c^9}} = 4.2217 \times 10^{-105}$ m^3
Planck momentum	$m_{Planck}c = \frac{\hbar}{L_{Planck}} = \sqrt{\frac{\hbar c^3}{G}} = 6.5249$ kg-m/s
Planck energy	$m_{Planck}c^2 = \frac{\hbar}{t_{Planck}} = \sqrt{\frac{\hbar c^5}{G}} = 1.9561 \times 10^9$ J At which quantum effects of gravity become strong.
Planck force	$\frac{m_{Planck}c^2}{L_{Planck}} = \frac{\hbar}{L_{Planck}\, t_{Planck}} = \frac{c^4}{G} = 1.2103 \times 10^{44}$ N • It is the gravitational attractive force of two bodies of **one Planck mass** each that are held **one Planck length** apart • It is the electrostatic attractive or repulsive force of **two Planck units of charges** that are held **one Planck length** apart.

Events happening at the Planck scale are undetectable with current scientific technology

Planck power	$\frac{m_{Planck}c^2}{t_{Planck}} = \frac{\hbar}{t^2_{Planck}} = \frac{c^5}{G} = 3.628 \times 10^{52}$ W
Planck density	$\frac{m_{Planck}}{L^3_{Planck}} = \frac{c^5}{\hbar G^2} = 5.1550 \times 10^{96}$ kg/m^3
Planck acceleration	$\frac{c}{t_{Planck}} = \sqrt{\frac{c^7}{\hbar G}} = 5.5608 \times 10^{51}$ m/s^2
Planck frequency	$\frac{1}{t_{Planck}} = \sqrt{\frac{c^5}{\hbar G}} = 1.8549 \times 10^{43}$ s^{-1}
Planck current	$\frac{q_{Planck}}{t_{Planck}} = \sqrt{\frac{4\pi\varepsilon_0 c^6}{G}} = 3.479 \times 10^{25}$ A
Planck voltage	$\frac{m_{Planck}c^2}{q_{Planck}} = \sqrt{\frac{c^4}{4\pi\varepsilon_0 G}} = 1.43 \times 10^{27}$ V

The density at which the Universe can no longer be described without quantum gravity

For energies approaching or exceeding $\sqrt{\frac{\hbar c^5}{G}} = 1.22 \times 10^{19}$ GeV, gravity is problematic and cannot be integrated with quantum mechanics. A new theory of quantum gravity is necessary. Approaches to this problem include:

- **String theory** (point-like particles are replaced by one-dimensional infinitesimal vibrating strings − smaller than atoms, electrons or quarks)
- **M-theory** (The Mother of all theories or Mystery − an 11 dimensional theory in which the weak and strong forces and gravity are unified and to which all the string theories belong)

A theory that extends general theory of relativity by quantizing spacetime—predicts that black holes evolve into white holes

- **Loop quantum gravity** (a theory of quantum gravity which aims to merge quantum mechanics and general theory of relativity)
- **Non-commutative geometry** (a branch of mathematics concerned with a geometric approach to noncommutative algebra)
- **Causal set theory** (an approach to quantum gravity that tries to replace the continuum spacetime structure of general relativity with the spacetime that has the property of discreteness and **causality**)

- The study of how things influence one other
- The study of how causes lead to effects

The idea of quantum foam arises out of Albert Einstein's idea that gravity is caused by the warping and curving of spacetime

Martin Bojowald

A German physicist who developed the application of loop quantum gravity to cosmology

The incorporation of a standard model into the framework of the quantum gravity

Loop Quantum Gravity (quantized space and time)	**String Theory**
Does not attempt to unify fundamental interactions	Attempts to unify all four fundamental interactions
Approaches the quantum gravity assuming the aspects of general relativity	Approaches the quantum gravity assuming the aspects of quantum theory
Does not require a super-symmetry	

Grand unification theory

↓

Fundamental symmetries existed at the beginning of the universe and then broke as the temperature dropped – just as H_2O which looks the same in every direction, freezes into ice, which has distinct directions.

Expanding matter

↓

White hole

↕ Quantum transition

Black hole

↑

Contracting matter

The Coulomb constant "$\frac{1}{4\pi\varepsilon_0}$" is a proportionality constant in electrostatics equations. It was named after the French physicist Charles-Augustin de Coulomb who introduced Coulomb's law.

Newton's law of gravitation:

$$F_G = \frac{Gm_1m_2}{r^2}$$

$$F_G \propto \frac{m_1m_2}{r^2}$$

G → Proportionality constant

$m_1 = m_2 = 1kg$

$r = 1m$

$$F_G = G$$

The universal gravitational constant is numerically equal to the Force of attraction between two unit masses placed at a unit distance apart.

Because $E=mc^2$:

$$F_G = \frac{GE_1E_2}{c^4r^2}$$

$$F_G \propto \frac{E_1E_2}{r^2}$$

$\frac{1}{F_{Planck}}$ → Proportionality constant

$E_1 = E_2 = 1J$

$r = 1m$

$$F_G = \frac{1}{F_{Planck}}$$

The reciprocal of Planck force is numerically equal to the Force of attraction between two unit energies placed at a unit distance apart.

$$\frac{(\text{Stoney mass})^2}{4\pi \times \hbar c \times \text{gravitoelectric gravitational constant}}$$

Fine structure constant: $\alpha = \frac{Z_0}{2R_K} = \frac{\text{impedance of free space}}{2 \times \text{von Klitzing constant}}$

$$\frac{(\text{Stoney mass})^2}{(\text{Planck mass})^2} = \frac{e^2}{q^2_{Planck}}$$

$$\text{Stoney mass} = \sqrt{\frac{e^2}{4\pi\varepsilon_0 G}}$$

$$\text{Stoney mass} = \text{Planck mass} \times \frac{\text{elementary charge}}{\text{Planck charge}}$$

$$\Delta x\ \Delta p \geq \frac{\hbar}{2}$$

$$\Delta E\ \Delta t \geq \frac{\hbar}{2}$$

$$\Delta x\ \Delta p \geq \frac{\text{Planck length} \times \text{Planck momentum}}{2}$$

$$\Delta E\ \Delta t \geq \frac{\text{Planck energy} \times \text{Planck time}}{2}$$

$$\frac{\Delta p}{\text{Planck momentum}} \geq \frac{\text{Planck length}}{\Delta x}$$

$$\frac{\Delta E}{\text{Planck energy}} \geq \frac{\text{Planck time}}{\Delta t}$$

Gravitoelectric gravitational constant: $\varepsilon_g = \frac{1}{4\pi G}$

Gravitomagnetic gravitational constant: $\mu_g = \frac{4\pi G}{c_g^2}$

The speed of gravitation:

$$c_g = \frac{1}{\sqrt{\varepsilon_g \mu_g}}$$

The Schwarzschild radius of the **Stoney mass**:

$$r_S = \frac{2Gm_S}{c^2} = \frac{2G}{c^2}\sqrt{\frac{e^2}{4\pi\varepsilon_0 G}}$$

$$\text{Planck force} = \frac{4\hbar c \ \times \text{fine structure constant}}{r_S^2}$$

Optical Telescope → **A telescope that is designed to collect visible light**

If we take the mass of electron as m, when it is moving with velocity v, then

$$m = \frac{m_e}{\sqrt{1-\frac{v^2}{c^2}}}$$

where m_e is the rest mass of the electron and m is the relativistic mass.

$$m^2 = \frac{m_e^2}{1-\frac{v^2}{c^2}}$$

If m = **Stoney mass =** $\sqrt{\frac{e^2}{4\pi\varepsilon_0 G}}$:

Hypernova → an exploding star that produces even more energy and light than a supernova

$$v = c\sqrt{1 - \frac{\text{Schwarzschild radius of electron}}{2 \times \text{Classical electron radius}}}$$

Velocity a electron must travel so that its relativistic mass to be equal to **Stoney mass**

The Compton wavelength of the **Stoney mass:**

$$\lambda_C = \frac{h}{m_S c} = \frac{h}{c} \times \sqrt{\frac{4\pi\varepsilon_0 G}{e^2}}$$

$$\lambda_C = \frac{2\pi \times \text{Planck length}}{\sqrt{\text{Fine structure constant}}}$$

The time it takes for a planet to complete one spin around its axis is called its **rotation period**.

The **Hawking radiation temperature** is:

$$T_{BH} = \frac{\hbar c^3}{8\pi G M k_B}$$

If M = **Stoney mass** = $\sqrt{\frac{e^2}{4\pi\varepsilon_0 G}}$:

$$T_{BH} = \frac{\hbar c^3}{8\pi G k_B}\sqrt{\frac{4\pi\varepsilon_0 G}{e^2}}$$

$$T_{BH} = \frac{\text{Planck energy}}{8\pi k_B \sqrt{\text{Fine structure constant}}}$$

$$T_{BH} = \frac{\text{Planck temperature}}{8\pi \sqrt{\text{Fine structure constant}}}$$

Observatory: A place where telescopes and other astronomical instruments are housed and used.

If a hot body were to reach the temperature of $\sqrt{\frac{\hbar c^5}{G k_B^2}}$, the radiation it would emit would have a wavelength of $\sqrt{\frac{\hbar G}{c^3}}$, at which quantum gravitational effects become relevant.

Planck temperature which equals about 100 million million million million million degrees, denoted by $T_{Planck} = \sqrt{\frac{\hbar c^5}{G k_B^2}}$, is the unit of temperature in the system of natural units known as Planck units. The Planck temperature is thought to be the upper limit of temperature that we know of according to the standard model of particle physics − which governs our universe.

In physics the Stoney units form a system of units named after the Irish physicist **George Johnstone Stoney**, who first proposed them in 1881

$$T_{Planck} = \sqrt{\frac{\hbar c^5}{G k_B^2}} = \frac{c_2}{2\pi L_{Planck}}$$

A fundamental limit of quantum theory in combination with gravitation − first introduced in 1899 by German physicist **Max Planck** together with his introduction of what today is known as the Planck length, the Planck mass and Planck time.

where: $L_{Planck} = \sqrt{\frac{\hbar G}{c^3}}$ is the Planck length and c_2 is the second radiation constant. **This means:**

$T_{Planck} \times L_{Planck}$ can never be less than or greater than $\frac{c_2}{2\pi}$ but $= \frac{c_2}{2\pi}$.

When the gold particles were smashed together, for a split second, the temperature reached 7.2 trillion degrees Fahrenheit. That was hotter than a supernova explosion. That was the hottest temperature that we have ever actually encountered in the **Large Hadron Collider** (the world's largest and most powerful particle accelerator).

Stoney length = $\sqrt{\frac{G e^2}{4\pi\varepsilon_0 c^4}} = \sqrt{\alpha} \times$ Planck length

$T_{Planck} \times$ **Stoney length** $= \frac{\sqrt{\alpha}\, c_2}{2\pi}$

The universe was at T_{Planck} about 10^{-43} seconds after the big bang explosion. At this time, the entire universe was roughly one-billionth of the diameter of a proton.

Planck density $\frac{c^5}{\hbar G^2}$ is very large – about equivalent to 10^{23} solar masses squeezed into the space of a single atomic nucleus. At Planck time after the Big Bang explosion, the cosmic mass density was thought to have been approximately 5.1550×10^{96} kg/m^3.

No temperature ⟶ **No heat exchange**.

Hagedorn temperature

{1.7×10^{12} K}

The temperature at which hadronic matter is no longer stable and must either "evaporate" or convert into quark matter – as such – it can be thought of as the "boiling point" of hadronic matter.

$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

- When the velocity of the particle v is very small compared to velocity of light c, then $\frac{v^2}{c^2}$ is negligible compared to one. Therefore,

$$m = m_0$$

- If the velocity of the particle v is comparable to the velocity of light c, then $\sqrt{1-\frac{v^2}{c^2}}$ is less than one, then

$$m > m_0$$

- If the velocity of a particle v is equal to velocity of light c, then it possesses infinite mass.

$$F_G = \frac{Gm_1m_2}{r^2}$$

$$F_G = \frac{m_1m_2}{m_{Planck}^2} \times \frac{\hbar c}{r^2}$$

$$m_1m_2 = m_{Planck}^2$$

- $m_1 = m_2 = m_{Planck}$
- $m_1 > m_{Planck}$ and $m_2 < m_{Planck}$

$$F_G = \frac{\hbar c}{r^2}$$

$$q_{Planck} = \frac{\text{elementary charge}}{\sqrt{\text{fine structure constant}}}$$

$$F_E = \frac{q_1q_2}{4\pi\varepsilon_0 r^2}$$

$$F_E = \frac{q_1q_2}{q_{Planck}^2} \times \frac{\hbar c}{r^2}$$

$$q_1q_2 = q_{Planck}^2 \longrightarrow F_E = \frac{\hbar c}{r^2}$$

- $q_1 = q_2 = q_{Planck}$
- $q_1 > q_{Planck}$ and $q_2 < q_{Planck}$

The rest mass energy of any particle is defined by the Albert Einstein's mass energy equivalence relation: $E_{rest} = m_0c^2 = k_BT_{threshold}$, where: m_0 is the mass of a stationary particle, also known as the invariant mass or the rest mass of the particle and $T_{threshold}$ implies the threshold temperature below which that particle is effectively removed from the universe. All particles have an intrinsic real internal vibration in their rest frame: $\upsilon_C = \frac{m_0c^2}{h} = \frac{c}{\lambda_C}$, where: υ_C and λ_C denote the quantum mechanical properties of a particle (i.e., the Compton frequency and Compton wavelength of the particle).

$$h\upsilon_C = \frac{hc}{\lambda_C} = k_BT_{threshold}$$

$$\lambda_C \times T_{threshold} = c_2$$

where: c_2 is the second radiation constant and is related to the **Stefan–Boltzmann constant** (also known as Stefan's constant) by: $\sigma = \frac{\pi^4 c_1}{15c_2^4}$. This means:

$$\lambda_C \propto \frac{1}{T_{threshold}}$$

The Compton wavelength of the particle is inversely proportional to the threshold temperature below which that particle is effectively removed from the universe.

$$T_{Planck} \times L_{Planck} = \frac{c_2}{2\pi}$$

$$T_{Planck} \times L_{Planck} = \frac{\lambda_C \times T_{threshold}}{2\pi}$$

$$(\lambda_C \times T_{threshold}) > (T_{Planck} \times L_{Planck})$$

$r_S \times \bar{\lambda}_C = 2 \times L_{Planck}^2 = 2 \times$ **Planck area**, where: $\bar{\lambda}_C = \frac{\hbar}{m_0c}$ is the reduced Compton wavelength of the particle. **This means:** The Schwarzschild radius of the particle times the reduced Compton wavelength of the particle is never smaller than a certain quantity, which is known as Planck area.

If the reduced Compton wavelength of the particle = **Stoney length**:

$$\frac{\hbar}{m_0 c} = \sqrt{\frac{Ge^2}{4\pi\varepsilon_0 c^4}}$$

$$m_0 = \underbrace{\frac{m_{Planck}}{\sqrt{\text{Fine structure constant}}}}$$

Mass a particle must possess so that its reduced Compton wavelength to be equal to **Stoney length**

$$E_{rest}^2 = \underbrace{m_0 c^2 \times h\upsilon_C}$$

$$E_{rest} = \text{Planck energy}\sqrt{\frac{\pi \times \text{Schwarzschild radius of the particle}}{\text{Compton wavelength of the particle}}}$$

$$E_{rest} = \text{Stoney mass} \times c^2 \sqrt{\frac{\pi \times \text{Schwarzschild radius of the particle}}{\text{Fine structure constant} \times \text{Compton wavelength of the particle}}}$$

Sunspot

A cooler region of the Solar surface – which looks dark in comparison to the hotter material around it.

If the Schwarzschild radius of the particle = **Stoney length**:

$$\frac{2Gm_0}{c^2} = \sqrt{\frac{Ge^2}{4\pi\varepsilon_0 c^4}}$$

$$m_0 = \underbrace{\frac{\sqrt{\text{Fine structure constant}} \times \text{Planck mass}}{2}}$$

Mass a particle must possess so that its Schwarzschild radius to be equal to **Stoney length**

Planetary Nebula →

A shell of gas ejected by a relatively low-mass star that is in the process of dying and becoming a white dwarf

Planck temperature = $\frac{m_{Planck} c^2}{k_B}$	Planck temperature = $\frac{m_S c^2}{\sqrt{\alpha}\, k_B}$
Planck area = L_{Planck}^2	Planck area = $\frac{L_S^2}{\alpha}$
Planck volume = L_{Planck}^3	Planck volume = $\frac{L_S^3}{\sqrt[3]{\alpha}}$
Planck energy = $\frac{\hbar}{t_{Planck}}$	Planck energy = $\frac{\hbar \sqrt{\alpha}}{t_S}$
Planck force = $\frac{m_{Planck} c^2}{L_{Planck}}$	Planck force = $\frac{m_S\, c^2}{L_S}$
Planck force = $\frac{\hbar}{L_{Planck}\, t_{Planck}}$	Planck force = $\frac{\alpha\, \hbar}{L_S\, t_S}$
Planck momentum = $m_{Planck}\, c$	Planck momentum = $\frac{m_S c}{\sqrt{\alpha}}$
Planck density = $\frac{m_{Planck}}{L_{Planck}^3}$	Planck density = $\frac{\alpha\, m_S}{L_S^3}$
Planck acceleration = $\frac{c}{t_{Planck}}$	Planck acceleration = $\frac{c \sqrt{\alpha}}{t_S}$
Planck frequency = $\frac{c}{L_{Planck}}$	Planck frequency = $\frac{c \sqrt{\alpha}}{L_S}$
Planck power = $\frac{m_{Planck} c^2}{t_{Planck}}$	Planck power = $\frac{m_S\, c^2}{t_S}$

- m_S = Stoney mass
- L_S = Stoney length
- t_S = Stoney time
- α = Fine structure constant

- $m_S = \frac{\text{elementary charge}}{\text{Planck charge}} \times \text{Planck mass}$
- $L_S = \frac{\text{elementary charge}}{\text{Planck charge}} \times \text{Planck length}$
- $t_S = \frac{\text{elementary charge}}{\text{Planck charge}} \times \text{Planck time}$

Astronomical transit is a phenomenon when a celestial body passes directly between a larger body and the observer.

PLANCK MASS: $m_{Planck} = \sqrt{\frac{\hbar c}{G}} = 2.17647 \times 10^{-8}$ kg, where: c is the speed of light in a vacuum, G is the gravitational constant, and ħ is the reduced Planck constant.

$$\frac{m_{Planck}}{m_0} = n$$ Number of particle masses that make up one Planck mass.

$$\frac{m_{Planck}c^2}{m_0 c^2} = \frac{k_B T_{Planck}}{k_B T_{threshold}} = n$$

$$T_{Planck} = n \times T_{threshold}$$

$$\lambda_C \times T_{threshold} = c_2 \qquad \lambda_C \times \frac{T_{Planck}}{n} = c_2$$

$$\lambda_C = \frac{c_2}{T_{Planck}} \times n \longrightarrow \lambda_C \propto n$$

This means: The Compton wavelength of the particle is directly proportional to the number of particle masses that make up one Planck mass.

$$\frac{\text{Planck charge}}{\text{Planck mass}} = \sqrt{4\pi\varepsilon_0 G} = \sqrt{\frac{\varepsilon_0}{\varepsilon_g}} = \sqrt{\frac{\text{Vacuum permittivity}}{\text{Gravitoelectric gravitational constant}}}$$

$$\frac{\text{electron charge}}{\text{electron mass}} = -1.75882001076 \times 10^{11} \text{ C/ kg}$$

$$\frac{\text{proton charge}}{\text{proton mass}} = +9.58 \times 10^{7} \text{ C/ kg}$$

When negatively charged electrons move in electric and magnetic fields the following two laws apply:

- $F = e\,(E + v \times B) \rightarrow$ **Lorentz force law**
- $F = m_e a = m_e \frac{dv}{dt} \rightarrow$ **Newton's second law of motion**

$$\frac{m_e}{e} = \frac{\text{electron mass}}{\text{electron charge}} = \frac{(E + v \times B)}{a}$$

- $F_{electric} = eE$
- $F_{magnetic} = eBv$

When equal: $v = \frac{E}{B}$

The **Planck length** $\approx 1.616255 \times 10^{-35}$ m is the scale at which classical ideas about gravity and space-time cease to be valid and quantum effects dominate.

Fine structure constant $= \frac{\mu_0 c}{2R_K} = \frac{\text{Vacuum permeability} \times \text{Planck speed}}{2 \times \text{von Klitzing constant}}$

Stoney mass $= \sqrt{\frac{\text{Vacuum permeability} \times \text{Planck speed}}{2 \times \text{von Klitzing constant}}} \times$ **Planck mass**

The **gravitational coupling constant** is a constant characterizing the gravitational attraction between a given pair of elementary particles. α_G is typically defined in terms of the gravitational attraction between two electrons. More precisely,

$\alpha_G = \frac{Gm_e^2}{\hbar c} = \frac{m_e^2}{m_{Planck}^2}$ where: m_e is the invariant mass of an electron

$$\alpha_G = \alpha \times \frac{m_e^2}{m_S^2}$$

- m_S = Stoney mass
- α = Fine structure constant

Number of electrons that make up one Planck mass $= \frac{m_{Planck}}{m_e} = \frac{1}{\sqrt{\alpha_G}}$

$$n = \frac{1}{\sqrt{\alpha_G}}$$

The Compton wavelength of electron:

$$\lambda_{C,e} = n \times \frac{c_2}{T_{Planck}}$$

$$\lambda_{C,e} = \frac{1}{\sqrt{\alpha_G}} \times \frac{c_2}{T_{Planck}}$$

$$\lambda_{C,e} \propto \frac{1}{\sqrt{\alpha_G}}$$

The Compton wavelength of the electron is inversely proportional to the square root of **gravitational coupling constant**.

Gravitational characteristic impedance of free space = $\frac{4\pi G}{c_g}$ → Speed of gravitation

Quantum of circulation: Half the ratio of the Planck constant to the mass of the electron.

$$Q_0 = \frac{h}{2m_e} = 3.636\ 947\ 5516 \times 10^{-4}\ m^2\ s^{-1}$$

$$E_{rest} = m_e c^2 = \frac{c_1}{4\pi Q_0}$$

c_1 = first radiation constant

$$E_{rest} \propto \frac{1}{Q_0}$$

The intrinsic energy of the electron is inversely proportional to the **Quantum of circulation**

$$Q_0 = \frac{h}{2m_e} = \frac{h}{2\sqrt{\alpha_G}\ m_{Planck}} = \sqrt{\frac{\alpha}{\alpha_G}} \times \frac{h}{2m_S}$$

- m_S = Stoney mass
- α = Fine structure constant

$\lambda_{C,e} = \frac{h}{m_e c}$ is the cutoff below which **quantum field theory** (which can describe particle creation and annihilation) becomes important.

$$\lambda_{C,e} = \frac{2h}{2m_e c} = \frac{2Q_0}{c} = 2Q_0 \sqrt{\varepsilon_0 \mu_0}$$

The **classical electron radius** is sometimes known as the **Compton radius** or the Lorentz radius or the Thomson scattering length is a combination of fundamental physical quantities that define a length scale for problems involving an electron interacting with electromagnetic radiation. The classical electron radius is defined by equating the electrostatic potential energy of a sphere of charge e and radius r_e with the intrinsic energy of the electron:

$$\frac{e^2}{4\pi\varepsilon_0 r_e} = m_e c^2$$

$$r_e = \frac{e^2}{4\pi\varepsilon_0 m_e c^2}$$ = Fine structure constant × reduced Compton wavelength of the electron

$$r_e = \frac{e^2}{4\pi\varepsilon_0\sqrt{\alpha_G}\, m_{Planck} c^2} = \frac{\alpha \times L_{Planck}}{\sqrt{\alpha_G}}$$

$$r_e = \sqrt{\frac{\alpha}{\alpha_G}} \times \textbf{Stoney length} = 2.8179 \times 10^{-15}\,\text{m}$$

$$\Delta x\, \Delta p \geq \frac{\hbar}{2}$$

$$\Delta x\, \Delta p \geq \frac{\lambda_C \times m_0 c}{2}$$

$$\frac{\Delta p}{m_0 c} \geq \frac{\lambda_C}{\Delta x}$$

For an electron, the **Thomson cross-section** is numerically given by:

$$\sigma_T = \frac{8\pi r_e^2}{3}$$

$$\sigma_T = \frac{8\pi}{3} \times \frac{\alpha}{\alpha_G} \times (\textbf{Stoney length})^2$$

Classical electron radius = Bohr radius × (Fine structure constant)^2

$$\text{Fine structure constant} = \sqrt{\frac{\text{Classical electron radius}}{\text{Bohr radius}}}$$

Bohr radius:

$$a_0 = \frac{4\pi\varepsilon_0\hbar^2}{m_e e^2} = \frac{\hbar}{m_e c\,\alpha} = \frac{\hbar}{\sqrt{\alpha_G}\, m_{Planck} c\,\alpha}$$

$$a_0 = \frac{L_{Planck}}{\sqrt{\alpha_G} \times \alpha}$$

The mean radius of the orbit of an electron around the nucleus of a **hydrogen atom** at its ground state (lowest-energy level)

$5.29177210903 \times 10^{-11}$ m

$$a_0 = \frac{\varepsilon_0\, h^2}{\pi\, m_e e^2} = \frac{\varepsilon_0}{\pi} \times 2\left(\frac{h}{2m_e}\right) \times \frac{h}{e^2}$$

$$a_0 = \frac{2\varepsilon_0 \times \text{Quantum of circulation} \times \text{von Klitzing constant}}{\pi}$$

$$a_0 = \frac{2 \times \text{Quantum of circulation} \times \text{von Klitzing constant}}{\pi\mu_0 c^2}$$

Wien's Displacement Law

$$\lambda_{Peak} \times T = b$$

The product of the **peak wavelength** and the temperature at which a blackbody radiates is constant − which means the peak of the radiation shifts to shorter wavelengths as the temperature increases.

Wien's constant: $b = \frac{hc}{4.9651 k_B} = \frac{c_2}{4.9651}$

$$c_2 = 4.9651\, b$$

The second radiation constant is 4.9651 times the Wien's constant

Radiation density constant:

$$a = \frac{4\sigma}{c} = \frac{8\pi^5 k_B^4}{15c^3h^3} = \frac{8\pi^5 k_B}{15c_2^3} = 7.5657 \times 10^{-16}\ \text{J m}^{-3}\ \text{K}^{-4}$$

$$a = \frac{4\sigma}{c} = \frac{4\pi^4 c_1}{15c_2^4}\sqrt{\mu_0 \varepsilon_0}$$

where: μ_0 is the absolute permeability of free space and ε_0 is the absolute permittivity of free space.

$$\frac{8\pi^5 k_B}{15c_2^3} = \frac{4\pi^4 c_1}{15c_2^4}\sqrt{\mu_0 \varepsilon_0}$$

$$k_B = \frac{c_1}{2\pi c_2}\sqrt{\mu_0 \varepsilon_0} = 1.3807 \times 10^{-23}\ \text{J/K}$$

$$k_B = \frac{c_1}{31.180\ b}\sqrt{\mu_0 \varepsilon_0}$$

Magnetic flux quantum:

$$\Phi_0 = \frac{h}{2e}$$

Conductance quantum:

$$G_0 = \frac{2e^2}{h}$$

where: e is the elementary charge.

$$\Phi_0 \times G_0 = e$$

$$\Phi_0 \times G_0 = \sqrt{\text{Fine structure constant}} \times q_{Planck}$$

$$\textbf{Planck charge} = \frac{\text{Magnetic flux quantum} \times \text{Conductance quantum}}{\sqrt{\text{Fine structure constant}}}$$

von Klitzing constant:

$$R_K = \frac{h}{e^2} = \frac{h}{\phi_0^2 G_0^2}$$

$$R_K = \frac{h}{e^2} = \frac{h}{\alpha \times q_{Planck}^2}$$

Conductance quantum:

$$G_0 = \frac{2e^2}{h} = \frac{2\alpha \times q_{Planck}^2}{h} = \frac{2}{R_K}$$

The magnetic coupling constant:

$$\beta = \frac{\varepsilon_0 hc}{2e^2} = \frac{\pi\hbar}{c\mu_0 e^2}$$

A **fundamental physical constant** characterizing the strength of the magnetic force interaction

$$\beta = \frac{\varepsilon_0 hc}{2e^2} = \frac{1}{4\alpha} = \frac{m_S^2}{4m_{Planck}^2} = \frac{L_S^2}{4L_{Planck}^2} = \frac{t_S^2}{4t_{Planck}^2}$$

Bohr radius is about 19,000 times bigger than the classical electron radius

$$\beta = \sqrt{\frac{\text{Bohr radius}}{16 \times \text{classical electron radius}}}$$

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \qquad c_g = \frac{1}{\sqrt{\mu_g \varepsilon_g}}$$

Time is relative

↓

It changes with speed and in the presence of gravity

$$\mu_0 \varepsilon_0 = \mu_g \varepsilon_g$$

$$\frac{\mu_0}{\mu_g} = \frac{\varepsilon_g}{\varepsilon_0}$$

If Gravity travel at the Speed of Light

The Planck charge $\sqrt{4\pi\varepsilon_0 \hbar c}$ is approximately 11.706 times greater than electron charge.

$$\Phi_0 \times G_0 \times R_K = \frac{h}{e}$$

Magnetic flux quantum × Conductance quantum × von Klitzing constant = Quantum / Charge Ratio

$$\Phi_0 \times G_0 \times R_K = \frac{h}{\sqrt{\text{Fine structure constant}} \times q_{\text{Planck}}}$$

$$\frac{\phi_0 \times G_0 \times R_K}{2Q_0} = \textbf{Electron mass-to-charge ratio}$$

Planck charge: $q_{\text{Planck}} = \sqrt{4\pi\varepsilon_0 \hbar c}$

$$q^2_{\text{Planck}} = 4\pi\varepsilon_0 \hbar c = 2h\sqrt{\frac{\varepsilon_0}{\mu_0}}$$

Planck conductance $= 4\pi\sqrt{\frac{\varepsilon_0}{\mu_0}}$

$$\frac{q_{\text{Planck}}}{t_{\text{Planck}}} \times q_{\text{Planck}} = 4\pi\sqrt{\frac{\varepsilon_0}{\mu_0}} \times \frac{\hbar}{t_{\text{Planck}}}$$

$$\text{Planck current} \times q_{\text{Planck}} = 4\pi\sqrt{\frac{\varepsilon_0}{\mu_0}} \times \text{Planck energy}$$

$$\text{Planck current} \times q_{\text{Planck}} = 4\pi\sqrt{\frac{\varepsilon_0}{\mu_0}} \times (q_{\text{Planck}} \times \text{Planck voltage})$$

$$\frac{1}{\text{Planck resistance}} = 4\pi\sqrt{\frac{\varepsilon_0}{\mu_0}}$$

Impedance of free space:

$$Z_0 = \mu_0 c = \sqrt{\frac{\mu_0}{\varepsilon_0}}$$

Admittance of free space:

$$Y_0 = \frac{1}{Z_0}$$

$$q_{Planck}^2 = \frac{2h}{Z_0} = 2h \times Y_0$$

$$q_{Planck}^2 = \frac{2R_K e^2}{Z_0}$$

$$q_{Planck} = e\sqrt{\frac{2R_K}{Z_0}} = \phi_0\, G_0\sqrt{\frac{2R_K}{Z_0}}$$

Stefan–Boltzmann law:

The radiative power of a black body is proportional to the surface area and to the fourth power of the black body's temperature

$$P = \varepsilon\sigma T^4 A$$

ε → Emissivity

- For all substances: $\varepsilon < 1$
- For a perfect black body: $\varepsilon = 1$

Stellar Planck constant:

$$h_S = 2 \times M \times R \times C_S$$

- M : mass of the neutron star
- R: radius of the neutron star
- C_S: the characteristic speed of the particles in the neutron star

Stellar Stefan–Boltzmann constant:

$$\Sigma_S = \frac{\text{Luminosity of the galaxy}}{\text{Area of the galaxy} \times (\text{Effective kinetic temperature of the stellar gas of the galaxy})^4}$$

Rydberg constant:

$$R_\infty = \underbrace{\frac{m_e e^4}{8\varepsilon_0^2 ch^3}} = \frac{\text{Fine structure constant}}{4\pi \times \text{Bohr radius}} = 10\,973\,731.6 \text{ m}^{-1}$$

$$R_\infty = \frac{1}{4}\sqrt{\frac{\mu_0}{\varepsilon_0}} \times \frac{\text{Fine structure constant}}{\text{von Klitzing constant} \times \text{Compton wavelength of the electron}}$$

Rydberg energy:

$$hc\,R_\infty = \frac{m_e c^2}{4}\sqrt{\frac{\mu_0}{\varepsilon_0}} \times \frac{\text{Fine structure constant}}{\text{von Klitzing constant}}$$

Rydberg frequency:

$$c\,R_\infty = \frac{\text{Compton frequency of the electron}}{4}\sqrt{\frac{\mu_0}{\varepsilon_0}} \times \frac{\text{Fine structure constant}}{\text{von Klitzing constant}}$$

Rydberg wavelength:

$$\frac{1}{R_\infty} = 4\sqrt{\frac{\varepsilon_0}{\mu_0}} \times \frac{\text{von Klitzing constant} \times \text{Compton wavelength of the electron}}{\text{Fine structure constant}}$$

Hartree energy:

$$E_h = 2R_\infty\,hc = \frac{m_e c^2}{2}\sqrt{\frac{\mu_0}{\varepsilon_0}} \times \frac{\text{Fine structure constant}}{\text{von Klitzing constant}} = 4.3597447222071 \times 10^{-18} \text{ J}$$

$$R_\infty = \frac{\text{Fine structure constant}}{4\pi \times \text{Bohr radius}}$$

$$\text{Fine structure constant} = \frac{e^2}{q_{\text{Planck}}^2} = 4\pi \times \text{Bohr radius} \times R_\infty$$

$$q_{\text{Planck}} = \frac{e}{\sqrt{4\pi \times \text{Bohr radius} \times R_\infty}}$$

$$R_\infty = \frac{\text{Fine structure constant}}{4\pi \times \text{Bohr radius}}$$

$$\text{Fine structure constant} = \sqrt{\frac{\text{Classical electron radius}}{\text{Bohr radius}}} = 4\pi \times \text{Bohr radius} \times R_\infty$$

$$R_\infty = \frac{1}{4\pi}\sqrt{\frac{\text{Classical electron radius}}{(\text{Bohr radius})^3}}$$

$$R_\infty = \frac{\text{Fine structure constant}}{4\pi \times \text{Bohr radius}}$$

$$\text{Fine structure constant} = \frac{\text{Conductance quantum} \times \text{impedance of free space}}{4} = 4\pi \times \text{Bohr radius} \times R_\infty$$

$$R_\infty = \frac{\text{Conductance quantum} \times \text{impedance of free space}}{16\pi \times \text{Bohr radius}}$$

$$\Delta S_0 + S_{BH} \geq 0$$

The sum of the entropy outside the black hole and the **total black hole entropy** never decreases and typically increases as a consequence of generic transformations of the black hole.

Nernst-Simon statement

↓

The entropy of a system at absolute zero temperature either vanishes or becomes independent of the intensive thermodynamic parameters

The **Bohr magneton** is defined in SI units by:

$$\mu_B = \frac{e\hbar}{2m_e} = \frac{\text{Faraday constant} \times \text{Planck angular momentum}}{2 \times \text{molar electron mass}} = \frac{\sqrt{\alpha} \times q_{Planck} \times Q_0}{2\pi}$$

↓

$9.27400968 \times 10^{-24} JT^{-1}$

$$\mu_B = \frac{\phi_0 \times G_0 \times Q_0}{2\pi}$$

Conductance quantum = $\frac{2\pi\ \mu_B}{\phi_0 \times G_0}$

The **Nuclear magneton** is defined in SI units by:

$$\frac{\mu_N}{\mu_B} = \frac{m_e}{m_p}$$

$$\mu_N = \frac{e\hbar}{2m_p} = \frac{\text{Faraday constant} \times \text{Planck angular momentum}}{2 \times \text{molar proton mass}} = 5.050783699 \times 10^{-27} JT^{-1}$$

$$\mu_N = \sqrt{\frac{\text{Fine structure constant}}{4\mu_0\varepsilon_0}} \times \text{Planck charge} \times \text{reduced Compton wavelength of proton}$$

Planck angular momentum = $m_{Planck} \times c \times L_{Planck} = \hbar$

Planck angular momentum = $\frac{m_S \times c \times L_S}{\alpha}$

Black Hole: A great amount of matter packed into a very small area where gravity is intense enough to prevent the escape of even the fastest moving particles. Not even light can break free.

Temperature → $T_{BH} = \dfrac{\hbar c^3}{8\pi G M k_B}$

$$\frac{T_{BH}}{T_{Planck}} = \frac{m_{Planck}}{8\pi M}$$

Evaporation time of a black hole:

$$t_{ev} = \frac{480c^2 V}{\hbar G}$$

$$\frac{t_{ev}}{t_{Planck}} = 480 \times \frac{V}{L^3_{Planck}}$$

Density → $\rho_{BH} = \dfrac{M}{\dfrac{4\pi r_S^3}{3}} = \dfrac{3c^6}{32\pi G^3 M^2}$

$$\frac{\rho_{BH}}{\rho_{Planck}} = \frac{m^2_{Planck}}{32\pi M^2}$$

where: $\rho_{Planck} = \dfrac{c^5}{\hbar G^2}$ is the Planck density.

If the star core's mass is more than about three times the mass of the Sun, the force of gravity overwhelms all other forces and produces a **black hole**.

The rate of evaporation energy loss of the black hole:

$$P = -\frac{dMc^2}{dt} = \frac{\hbar c^6}{15360\pi G^2 M^2}$$

$$\frac{P}{P_{Planck}} = \frac{m^2_{Planck}}{15360\pi M^2}$$

where: $P_{Planck} = \dfrac{c^5}{G}$ is the Planck power.

Entropy → $S_{BH} = \dfrac{4\pi k_B M^2}{m^2_{Planck}}$

$$\frac{S_{BH}}{S_{Planck}} = \frac{4\pi M^2}{m^2_{Planck}}$$

where: $S_{Planck} = k_B$ is the Planck entropy.

If M = $\sqrt{\frac{\hbar c}{G}}$ → Planck mass:

- $T_{BH} = \frac{T_{Planck}}{8\pi}$
- $\rho_{BH} = \frac{\rho_{Planck}}{32\pi}$
- $P = \frac{P_{Planck}}{15360\pi}$
- $S_{BH} = 4\pi \times S_{Planck}$

If V = L^3_{Planck} → Planck volume:

$$t_{ev} = 480 \times t_{Planck} = \frac{480 t_S}{\sqrt{\alpha}}$$

Compton shift:

$$\Delta\lambda = \frac{h}{m_e c}(1-\cos\theta)$$

If $\Delta\lambda$ = Stoney length:

$$\sqrt{\alpha} \times L_{Planck} = \frac{h}{m_e c}(1-\cos\theta)$$

$$\theta = \cos^{-1}\left(1 - \frac{\sqrt{\alpha_G \alpha}}{2\pi}\right)$$

The wavelength shift of the scattered photon in an angle of $\theta = \cos^{-1}\left(1 - \frac{\sqrt{\alpha_G \alpha}}{2\pi}\right)$ is equal to the **Stoney length**.

Second radiation constant: $c_2 = \frac{2\pi \hbar c}{k_B} = \frac{2\pi \times \text{Planck angular momentum} \times \text{Planck speed}}{\text{Planck entropy}}$

If $\Delta\lambda$ = classical electron radius:

$$\frac{e^2}{4\pi\varepsilon_0 m_e c^2} = \frac{h}{m_e c}(1-\cos\theta)$$

$$\theta = \cos^{-1}\left(1-\frac{\alpha}{2\pi}\right)$$

The wavelength shift of the scattered photon in an angle of $\theta = \cos^{-1}\left(1-\frac{\alpha}{2\pi}\right)$ is equal to the **Classical electron radius** .

If $\Delta\lambda$ = Bohr radius:

$$\frac{4\pi\varepsilon_0 \hbar^2}{m_e e^2} = \frac{h}{m_e c}(1-\cos\theta)$$

$$\theta = \cos^{-1}\left(1-\frac{1}{2\pi\alpha}\right)$$

The wavelength shift of the scattered photon in an angle of $\theta = \cos^{-1}\left(1-\frac{1}{2\pi\alpha}\right)$ is equal to the **Bohr radius** .

First radiation constant: $c_1 = 4\pi^2\hbar c^2$

$c_1 = 4\pi^2$ × Planck angular momentum × (Planck speed) ^{2}

Spin-statistics connection theorem:

- Fermions (such as electrons and protons) having a half integer spin must be described by **Fermi-Dirac statistics**
- Bosons (such as photons and helium-4 atoms) having an integer spin must be described by **Bose-Einstein statistics.**

The time it takes for light to travel a distance equal to $\frac{2GM}{c^2}$:

$$\tau_1 = \frac{2GM}{c^2} \times \frac{1}{c}$$

$$E = \frac{P_{Planck}}{2} \times \tau_1$$

where: E is the energy of the black hole and $P_{Planck} = \frac{c^5}{G}$ is the Planck power.

The time it takes for light to travel a distance equal to **Stoney length**:

$$\tau_2 = \frac{L_S}{c} = \frac{\sqrt{\alpha} \times L_{Planck}}{c}$$

$$\tau_2 = \sqrt{\alpha} \times t_{Planck}$$

The time it takes for light to travel a distance equal to $\frac{h}{m_e c}$:

$$\tau_3 = \frac{h}{m_e c} \times \frac{1}{c} = \frac{h}{m_e c^2} = \frac{1}{\upsilon_C}$$

$$\tau_3 = \frac{h}{m_e c^2} = 2Q_0 \times \mu_0 \times \varepsilon_0$$

$$c_1 = 2\pi hc^2$$

$$c_2 = \frac{hc}{k_B}$$

$$\frac{c_1}{c_2} = 2\pi c k_B$$

$$\frac{c_1}{c_2} = 2\pi \times \text{Planck speed} \times \text{Planck entropy}$$

Unruh temperature:

$$T_U = \frac{\hbar a}{2\pi k_B c}$$

where: ħ is the reduced Planck constant, a is the local acceleration, c is the speed of light and k_B is the Boltzmann constant.

- a of 2.47×10^{20} m/s^2 corresponds approximately to a T_U of 1 K.
- a of 1 m/s^2 corresponds approximately to a T_U of 4.06×10^{-21} K.

$$T_U = \frac{\hbar a\, c_2}{c_1} = \frac{\text{Planck angular momentum} \times a \times c_2}{c_1}$$

Hawking–Unruh temperature:

$$T_H = \frac{\hbar g}{2\pi k_B c}$$

where: g is the surface gravity of a black hole.

$$T_H = \frac{c_2\, g \sqrt{\mu_0 \varepsilon_0}}{4\pi^2}$$

PCT theorem

All interactions are invariant under the Charge, parity and time reversal symmetry

The vacuum energy density or dark energy density is defined as:

$$\varepsilon_\Lambda = \frac{c^4}{8\pi G} \times \Lambda$$

Λ = cosmological constant

The mass density corresponding to the vacuum energy density is expressed as:

$$\rho_\Lambda = \frac{\varepsilon_\Lambda}{c^2}$$

The act of tearing space apart resulting in a sort of "reverse singularity" − where space and time can either be reborn or can disappear into nothingness.

If dark energy gets stronger and stronger over time, it will eventually overcome gravitational force of attraction and then everything is torn apart.

Big Rip

The **ultimate fate of the universe** − in which the matter of the universe and even the fabric of spacetime itself − is progressively torn apart by the expansion of the universe at a certain time in the future − until distances between single atoms will become infinite.

Dark energy

- the cosmological constant from **General theory of Relativity**
- the zero-point energy inherent to space from quantum field theory

maintains a constant energy density and would cause all galaxies to recede from each other at speeds proportional to their distance of separation.

h, c	Quantum Field Theory and the standard model of particle physics
G, c	General Theory of Relativity (**geometric theory of gravitation**) and the standard model of cosmology
h, k_B	Quantum Statistics and Modern quantum physics

Second radiation constant:

$$c_2 = \frac{hc}{k_B} = \frac{N_A\, h}{N_A\, k_B} \times c$$

- N_A = **Avogadro number** (the number of particles that are contained in one mole of a substance)

  $\left\{ 6.02214076 \times 10^{23} \right\}$

$$c_2 = \frac{\text{Molar Planck constant}}{\text{Ideal gas constant}} \times \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

$$\frac{F}{R} = \frac{N_A\, e}{N_A\, k_B} = \frac{\textbf{Molar electron charge}}{\text{Ideal gas constant}}$$

$$\frac{F}{R} = \frac{e}{hc} \times c_2$$

$$\frac{F}{R} = \frac{K_J}{2} \times c_2 \sqrt{\mu_0 \varepsilon_0}$$

where: F is the Faraday constant and K_J is the Josephson constant.

Quantum of circulation $= \dfrac{h}{2m_e} = \dfrac{\text{Molar Planck constant}}{2 \times \text{Molar electron mass}}$

The Avogadro number is named after the Italian scientist **Amedeo Avogadro** – who − in 1811 − first proposed that the equal volumes of gases under the same conditions of temperature and pressure will contain equal numbers of molecules.

$$Q_0 \times r_S = \frac{h}{2m_e} \times \frac{2Gm_e}{c^2}$$

$$Q_0 \times r_S = 2\pi \frac{G\hbar}{c^2} = 2\pi \frac{\text{Planck volume}}{\text{Planck time}}$$

$$\textbf{Planck volumetric flow rate} = \frac{Q_0 \times r_S}{2\pi}$$

$$\frac{\text{Energy}}{\text{mass}} = \text{Specific energy}$$

$$\frac{\text{Planck Energy}}{\text{Planck mass}} = \text{Planck Specific energy} = c^2$$

Planck specific energy = (Planck speed) ^{2}

Hawking radiation temperature:

$$T_{BH} = \frac{\hbar c^3}{8\pi G M k_B} = 6 \times 10^{-8}\ \text{K}\ \frac{\text{Solar mass}}{\text{Mass of the black hole}}$$

Unruh temperature:

Black hole's gravitational acceleration

$$T_U = \frac{\hbar a}{2\pi k_B c}$$

If $a = \frac{c^4}{4GM}$:

$$T_U = T_{BH}$$

The temperature of the vacuum − observed by an isolated observer accelerating at the Black hole's gravitational acceleration of $g = \frac{c^4}{4GM}$ m/s^2 is **Hawking radiation temperature**.

$$r_S \times T_{BH} = \underbrace{\frac{2GM}{c^2} \times \frac{\hbar c^3}{8\pi G M k_B}}$$

$$r_S \times T_{BH} = \frac{c_2}{8\pi^2}$$

This means: $r_S \times T_{BH}$ can never be less than or greater than $\frac{c_2}{8\pi^2}$ but = $\frac{c_2}{8\pi^2}$.

Unruh temperature = $\frac{\hbar a}{2\pi k_B c}$

If **Unruh temperature = Planck temperature:**

$$T_{Planck} = \frac{\hbar a}{2\pi k_B c} \longrightarrow \boxed{a = 2\pi \times a_{Planck}}$$

If **a = Planck acceleration:**

$$T_U = \frac{\hbar a_{Planck}}{2\pi k_B c} \longrightarrow \boxed{T_U = \frac{T_{Planck}}{2\pi}}$$

- **Josephson constant:** $K_J = \frac{2e}{h}$
- **Magnetic flux quantum:** $\phi_0 = \frac{h}{2e}$

$K_J \times \phi_0 = 1$

- **Conductance quantum:** $G_0 = \frac{2e^2}{h}$
- **Resistance quantum:** $R_0 = \frac{h}{2e^2}$

$G_0 \times R_0 = 1$

Modified Newtonian dynamics

Hypothesis proposing a modification of Newton's law of universal gravitation to account for observed properties of galaxies

Schwarzschild radius of electron:

$$r_S = \frac{2Gm_e}{c^2}$$

The threshold temperature below which the electron is effectively removed from the universe:

$$T_{threshold} = \frac{m_e c^2}{k_B}$$

$$r_S \times T_{threshold} = \frac{2Gm_e^2}{k_B}$$

$$r_S \times T_{threshold} = \frac{\alpha_G \times c_2}{\pi}$$

Irradiation

↓

The process by which an object is exposed to radiation

$$KE = e \times V$$

$$KE = \sqrt{\alpha} \times q_{Planck} \times V$$

$$\frac{KE}{E_{Planck}} = \sqrt{\alpha} \times \frac{V}{V_{Planck}}$$

If V = Planck voltage:

$$KE = \sqrt{\alpha} \times E_{Planck}$$

Planck voltage:

$$V_{Planck} = \frac{\text{Planck energy}}{\text{Planck charge}} = \sqrt{\frac{c^4}{4\pi\varepsilon_0 G}} = \sqrt{\text{Planck force} \times \text{Coulomb constant}}$$

Planck current:

$$I_{Planck} = \frac{\text{Planck charge}}{\text{Planck time}} = \sqrt{\frac{4\pi\varepsilon_0 c^6}{G}} = \sqrt{\frac{\text{Planck force} \times \text{Planck specific energy}}{\text{Coulomb constant}}}$$

Planck pressure:

$$\Pi_{\text{Planck}} = \frac{\text{Planck force}}{\text{Planck area}} = \frac{c^7}{\hbar G^2} = \frac{\hbar}{L_{\text{Planck}}^3\, t_{\text{Planck}}} = \frac{\alpha^2 \hbar}{L_S^3\, t_S} = 4.633 \times 10^{113}\ \text{Pa}$$

Most of the matter in the Universe is dark

Dark Matter → nonluminous and it looks like a matter

Why does it gravitate as ordinary matter does, and thus slows the expansion of the universe?

$$\Pi_{\text{Planck}} = \frac{\alpha^2 \hbar}{L_S^3\, t_S}$$

$$\Pi_{\text{Planck}} = \frac{\text{classical electron radius}}{\text{Bohr radius}} \times \frac{\hbar}{L_S^3\, t_S}$$

$$\text{Planck acceleration} = \frac{\text{Planck frequency}}{\sqrt{\varepsilon_0 \mu_0}}$$

$$\lambda_{C,e} = \frac{h}{m_e c} = \mathbf{2 \times Magnetic\ flux\ quantum} \times \text{Electron Charge to mass ratio} \times \sqrt{\mu_0 \varepsilon_0}$$

Both Albert Einstein's and Sir Isaac Newton's theories of gravitation have a problem when they encounter quantum mechanics and that problem involves the very nature of space and time.

$$\sqrt{\frac{\hbar G}{c^3}} = L_{Planck} \rightarrow \text{a fundamental limit to space}$$

$$\sqrt{\frac{\hbar G}{c^5}} = t_{Planck} \rightarrow \text{a fundamental limit to time}$$

$$S = k_B \ln W$$

A measure of statistical disorder of a system

This equation takes pride of place on the **Ludwig Eduard Boltzmann's grave** in the Zentralfriedhof, Vienna.

If S = Planck entropy = k_B:

$$W = e = 2.718281828459045$$

$$\text{Planck capacitance} = \frac{\text{Planck charge}}{\text{Planck voltage}} = \sqrt{4\pi\hbar c\varepsilon_0} \times \sqrt{\frac{4\pi\varepsilon_0 G}{c^4}}$$

Planck capacitance = Coulomb constant × Planck length

↓

$$\frac{1}{4\pi\varepsilon_0} = \frac{\mu_0 c^2}{4\pi} = \frac{\mu_0}{4\pi} \times \text{Planck specific energy} = \frac{\mu_0}{4\pi} \times (\text{Planck speed})^2$$

$$Q = n_e \times e$$

$$\frac{dQ}{dt} = \frac{dn_e}{dt} \times e$$

$$I = \frac{dn_e}{dt} \times e$$

"The infinite is nowhere to be found in reality, no matter what experiences, observations, and knowledge are appealed to"

– David Hilbert

GM_{sun}

Heliocentric gravitational constant

If $I = \text{Planck current} = \frac{\text{Planck charge}}{\text{Planck time}}$:

$$\frac{q_{Planck}}{t_{Planck}} = \frac{dn_e}{dt} \times e$$

$$\frac{dn_e}{dt} = \frac{1}{\sqrt{\alpha}\, t_{Planck}}$$

$$\text{Rate of flow of electrons} = \frac{1}{\text{Stoney time}}$$

Standard gravitational parameter:

$$\mu = GM$$

For Planck mass:

$$\mu = Gm_{Planck} = \sqrt{G \times \hbar \times c}$$

For Stoney mass:

$$\mu = Gm_S = \sqrt{\alpha}\ Gm_{Planck} = \sqrt{\alpha \times G \times \hbar \times c}$$

Classical electron radius:

$$r_e = \frac{1}{4\pi\varepsilon_0 m_e c^2} \times e^2$$

The threshold temperature below which the electron is effectively removed from the universe:

$$T_{threshold} = \frac{m_e c^2}{k_B}$$

$$r_e \times T_{threshold} = \frac{\alpha \times c_2}{2\pi}$$

Bohr radius:

$$a_0 = \frac{\hbar}{m_e c\alpha}$$

$$a_0 \times T_{threshold} = \frac{c_2}{2\pi\alpha}$$

Black Hole Density:

$$\rho_{BH} = \rho_{Planck} \times \frac{m_{Planck}^2}{32\pi M^2} = \rho_{Planck} \times \frac{m_S^2}{32\pi\ \alpha\ M^2}$$

If $M = m_S = \sqrt{\frac{e^2}{4\pi\varepsilon_0 G}}$:

$$\rho_{BH} = \frac{\rho_{Planck}}{32\pi\ \alpha}$$

The rate of evaporation energy loss of the black hole:

$$P = P_{Planck} \times \frac{m_{Planck}^2}{15360\pi M^2} = P_{Planck} \times \frac{m_S^2}{15360\ \pi\ \alpha\ M^2}$$

If $M = m_S = \sqrt{\frac{e^2}{4\pi\varepsilon_0 G}}$:

$$P = \frac{P_{Planck}}{15360\ \pi\ \alpha}$$

Black hole Entropy: $S_{BH} = S_{Planck} \times \frac{4\pi M^2}{m_{Planck}^2} = \frac{4\pi\ \alpha\ M^2}{m_S^2}$

If $M = m_S = \sqrt{\frac{e^2}{4\pi\varepsilon_0 G}}$:

$$S_{BH} = S_{Planck} \times 4\pi\ \alpha$$

$$P \times S_{BH} = P_{Planck} \times \frac{m_{Planck}^2}{15360\pi M^2} \times S_{Planck} \times \frac{4\pi M^2}{m_{Planck}^2}$$

$$P \times S_{BH} = \frac{\text{Planck power} \times \text{Planck entropy}}{3840} = \frac{k_B c^5}{3840G}$$

$$\rho_{BH} \times S_{BH} = \rho_{Planck} \times \frac{m_{Planck}^2}{32\pi M^2} \times S_{Planck} \times \frac{4\pi M^2}{m_{Planck}^2}$$

$$\rho_{BH} \times S_{BH} = \frac{\text{Planck density} \times \text{Planck entropy}}{8} = \frac{k_B c^5}{8\hbar G^2}$$

Rydberg wavelength:

$$\lambda_{R_\infty} = \frac{1}{\text{Rydberg constant}} = \frac{8\varepsilon_0^2 h^3 c}{m_e e^4}$$

$$\lambda_{R_\infty} \times T_{threshold} = \frac{8\varepsilon_0^2 h^3 c}{m_e e^4} \times \frac{m_e c^2}{k_B}$$

$$\lambda_{R_\infty} \times T_{threshold} = \frac{2c_2}{\alpha^2}$$

$$r_S \times r_e = \frac{2Gm_e}{c^2} \times \frac{1}{4\pi\varepsilon_0 m_e c^2}$$

$$r_S \times r_e = 2\alpha L_{Planck}^2$$

$$\left\{ r_S \times r_e = 2 \times \text{Fine structure constant} \times \text{Planck area} \right\}$$

$$r_S \times a_0 = \frac{2Gm_e}{c^2} \times \frac{\hbar}{m_e c\alpha}$$

$$r_S \times a_0 = \frac{2L_{Planck}^2}{\alpha}$$

$$\left\{ r_S \times a_0 = \frac{2 \times \text{Planck area}}{\text{Fine structure constant}} \right\}$$

$$r_S \times r_e = 2L_S^2$$

$$r_S \times a_0 = \frac{2L_S^2}{\alpha^2}$$

Mach's Principle

The inertia of the mass is caused by all other masses in the entire universe

$$\lambda_{R_\infty} \times r_S = \frac{8\varepsilon_0^2 h^3 c}{m_e e^4} \times \frac{2Gm_e}{c^2}$$

$$\lambda_{R_\infty} \times r_S = \frac{8\pi L_{Planck}^2}{\alpha^2}$$

$$\lambda_{R_\infty} \times r_S = \frac{8\pi \times \text{Planck area}}{\alpha^2}$$

$$\lambda_{R_\infty} \times r_S = \frac{8\pi L_S^2}{\alpha^3}$$

Science aims at constructing a world which shall be symbolic of the world of commonplace experience.

– Arthur Eddington

- **Stoney length = L_S =** $\sqrt{\frac{Ge^2}{4\pi\varepsilon_0 c^4}} = e\sqrt{\frac{\text{Coulomb constant}}{\text{Planck force}}}$

- **Stoney time = T_S =** $\sqrt{\frac{Ge^2}{4\pi\varepsilon_0 c^6}} = e\sqrt{\frac{\mu_0}{4\pi \times \text{Planck force}}}$

$$c = \frac{L_{Planck}}{t_{Planck}} = \frac{L_S}{t_S} = \sqrt{\text{Planck specific energy}}$$

Refractive index: $n = \frac{c}{v} = \frac{1}{v\sqrt{\varepsilon_0 \mu_0}} = \frac{L_S}{v \times t_S} = \frac{\text{Planck speed}}{v}$

First radiation constant:

$$c_1 = 4\pi^2 \times \text{Planck angular momentum} \times \frac{(\text{Stoney length})^2}{(\text{Stoney time})^2}$$

Bohr's Quantization Rule:

$$L = n\hbar$$

$$n = \frac{\text{electron angular momentum}}{\text{Planck angular momentum}}$$

For n = 1:

Electron angular momentum = Planck angular momentum

Second radiation constant:

$$c_2 = \frac{hc}{k_B} = \frac{\text{molar Planck constant}}{\text{Ideal gas constant}} \times \frac{\text{Stoney length}}{\text{Stoney time}}$$

Radiation Constant = 4 × Stefan-Boltzmann constant × $\frac{\text{Stoney time}}{\text{Stoney length}}$

Black hole temperature: $T_{BH} = \frac{\hbar c^3}{8\pi G k_B m_0}$

The threshold temperature below which the particle of mass m_0 is effectively removed from the universe:

$$T_{threshold} = \frac{m_0 c^2}{k_B}$$

$$T_{BH} \times T_{threshold} = \frac{T_{Planck}^2}{8\pi}$$

Kardashev scale

Classification of **alien civilization** based on how much energy an extraterrestrial civilization uses

- **Type I civilization** (planetary civilization)**:** A civilization capable of using and storing all of the energy resources available on its planet.
- **Type II civilization** (stellar civilization)**:** A civilization capable of using and controlling all of the energy resources available in its planetary system or all of the energy that its star emits.
- **Type III civilization** (galactic civilization)**:** A civilization capable of accessing and controlling all of the energy resources available in its galaxy.

$$Q_0 \times T_{\text{threshold}} = \frac{h}{2m_e} \times \frac{m_e c^2}{k_B}$$

$$Q_0 \times T_{\text{threshold}} = \frac{c_2}{\sqrt{4\mu_0\varepsilon_0}} = \frac{c_2 L_S}{2t_S}$$

White's Energy Formula:

C = E × T

- E is a measure of energy consumed per capita per year
- T is the measure of efficiency of technical factors utilizing the energy
- C represents the degree of cultural development

Culture evolves as the amount of energy harnessed per capita per year is increased

$$a \propto t^{\frac{2}{3(1+w)}}$$

$$\rho \propto a^{-3(1+w)}$$

- **Radiation dominated universe** (w = $\frac{1}{3}$) :

$$a \propto t^{\frac{1}{2}}$$

$$\rho \propto a^{-4}$$

Planetary engineering

↓

The development and application of technology for the purpose of influencing the environment of a planet

- **Non-relativistic matter dominated universe** (w = 0):

$$a \propto t^{\frac{2}{3}}$$

$$\rho \propto a^{-3}$$

- **Dark energy dominated universe** (w = −1):

$$a \propto e^{Ht} \text{ with } H = \sqrt{\frac{\Lambda}{3}}$$

Terraforming

The hypothetical process of deliberately modifying the Planet's atmosphere, temperature, surface topography or ecology to be similar to those of Earth in order to make it suitable for human life

The gravitational force between 2 electrons is:

$$F_G = \frac{Gm_e^2}{r^2}$$

The electrical force between 2 electrons is:

$$F_E = \frac{e^2}{4\pi\varepsilon_0 r^2}$$

Geoengineering

Planetary engineering applied to Earth

$\beta \rightarrow$ magnetic coupling constant

$$\frac{F_G}{F_E} = \frac{\alpha_G}{\alpha} = 4\beta \times \alpha_G$$

The electric field **E** is related to the electric force **F** that acts on an electron charge **e** by:

$$E = \frac{F}{e}$$

$$F = \sqrt{\alpha}\, q_{Planck}\, E$$

- **Habitable Planet:** A Planet with an environment hospitable to life.
- **Biocompatible Planet:** A Planet possessing the necessary physical parameters for life to flourish on its surface.

$$\mu_B \times T_{threshold} = \frac{e\hbar}{2m_e} \times \frac{m_e c^2}{k_B} = \frac{e}{4\pi} \times c_2 \times c$$

$$\mu_B \times T_{threshold} = \frac{e}{4\pi} \times c_2 \times \sqrt{\frac{c_1}{2\pi h}}$$

$$\mu_B \times T_{threshold} = c_2 \sqrt{\frac{c_1}{32\pi^3 R_K}}$$

	If $M = m_e$:
$T_{BH} = T_{Planck} \times \dfrac{m_{Planck}}{8\pi M}$	$T_{BH} = T_{Planck} \times \dfrac{m_{Planck}}{8\pi m_e}$ $T_{BH} = \dfrac{T_{Planck}}{8\pi\sqrt{\alpha_G}}$
$\rho_{BH} = \rho_{Planck} \times \dfrac{m^2_{Planck}}{32\pi M^2}$	$\rho_{BH} = \rho_{Planck} \times \dfrac{m^2_{Planck}}{32\pi m_e^2}$ $\rho_{BH} = \dfrac{\rho_{Planck}}{32\pi\alpha_G}$
$P = P_{Planck} \times \dfrac{m^2_{Planck}}{15360\pi M^2}$	$P = P_{Planck} \times \dfrac{m^2_{Planck}}{15360\pi m_e^2}$ $P = \dfrac{P_{Planck}}{15360\pi\alpha_G}$
$S_{BH} = S_{Planck} \times \dfrac{4\pi M^2}{m^2_{Planck}}$ $\alpha_G \rightarrow$ Gravitational coupling constant	$S_{BH} = S_{Planck} \times \dfrac{4\pi m_e^2}{m^2_{Planck}}$ $S_{BH} = 4\pi\ S_{Planck}\ \alpha_G$

$v \times v_{Phase} = c^2 = (\text{Planck speed})^2 = \text{Planck specific energy}$

Since the particle speed $v < c$ for any particle that has mass − according to **Albert Einsteinian special theory of relativity**, the phase velocity of matter waves always exceeds c, i.e. v_{Phase} > Planck speed

$$v_{Phase} > \frac{\text{Planck length}}{\text{Planck time}}$$

- $v_{Phase} > \dfrac{\text{Stoney length}}{\text{Stoney time}}$
- $v < \dfrac{\text{Stoney length}}{\text{Stoney time}}$

The strong coupling constant

{ One of the fundamental parameters of the Standard Theory of particle physics that defines the strength of the force that holds protons and neutrons together }

The **electrostatic repulsion** between 2 electrons is described in quantum electrodynamics as the result of an exchange of a virtual photon between the 2 electrons.

A particle with a mass m, when at motion, has an energy of E = $\sqrt{p^2c^2 + m_0^2c^4}$. But for photons E = $\sqrt{p^2c^2 + 0}$ = pc since they are never at rest; they always move at the speed of light.

→ Energy ∝ momentum } Requirement for masslessness

$m_ec^2 = \frac{e^2}{4\pi\varepsilon_0 r}$	r = **Classical electron radius** = $\frac{e^2}{4\pi\varepsilon_0 m_e c^2}$
$m_ec^2 = \frac{Gm_e^2}{r}$	r = $\frac{\textbf{Schwarzschild radius of electron}}{2} = \frac{Gm_e}{c^2}$

1 eV is the energy that an electron acquires when it is accelerated through a voltage of one volt.

m_e × Hartree velocity × Bohr radius = ħ {**Planck angular momentum**}

1 keV = 1000 eV
1 MeV = 1000 keV
1 GeV = 1000 MeV
1 TeV = 1000 GeV

Rydberg constant sets the magnitude of the various allowed electron energy levels in atoms such as hydrogen.

At energy of **14,000 GeV** (i.e., 15,000 times the mass of a proton in units of energy):

The velocity of the proton is 0.999999998c (so almost equal to c).

$$m_p c^2 = \frac{G m_e^2}{r}$$

$$r = \frac{\mu_N}{\mu_B} \times \frac{r_S}{2}$$

Distance between 2 electrons at which gravitational potential energy between them is equal to intrinsic energy of proton

$$m_{Planck} c^2 = \frac{G m_e^2}{r}$$

$$r = \sqrt{\text{electron gravitational coupling constant}} \times \frac{r_S}{2}$$

Distance between 2 electrons at which gravitational potential energy between them is equal to Planck energy

My studies of the natural sciences have particularly involved that part of physics which looks at the atomic world.

Amedeo Avogadro

	M	$\mathbf{\frac{2GM}{c^2}}$
Sun	1.99×10^{30} kg	2.95×10^{3} m
Jupiter	1.90×10^{27} kg	2.82 m
Earth	5.97×10^{24} kg	8.87×10^{-3} m
Moon	7.35×10^{22} kg	1.09×10^{-4} m
Saturn	5.683×10^{26} kg	8.42×10^{-1} m
Uranus	8.681×10^{25} kg	1.29×10^{-1} m
Neptune	1.024×10^{26} kg	1.52×10^{-1} m
Mercury	3.285×10^{23} kg	4.87×10^{-4} m
Venus	4.867×10^{24} kg	7.21×10^{-3} m
Mars	6.39×10^{23} kg	9.47×10^{-4} m
Human	70 kg	1.04×10^{-25} m
Planck mass	2.18×10^{-8} kg	3.23×10^{-35} m (Twice the Planck length)

Stellar gas constant = Avogadro constant × Stellar Boltzmann constant

The relativistic energy of an electron can be expressed in terms of its momentum in the expression:

$$E = \sqrt{p^2c^2 + m_e^2c^4}$$

$$E = \sqrt{p^2c^2 + \alpha_G E_{Planck}^2}$$

If p = Planck momentum = $\sqrt{\frac{\hbar c^3}{G}}$:

$$E = E_{Planck}\sqrt{1 + \alpha_G}$$

Relativistic energy a electron must possess so that its **momentum** to be equal to Planck momentum

Stoney energy:

$$E_S = m_S c^2 = \sqrt{\alpha}\,\frac{m_{Planck}L_{Planck}^2}{t_{Planck}^2}$$

$$\frac{m_S L_S^2}{t_S^2}$$

Stoney temperature:

$$T_S = \frac{E_S}{k_B} = \sqrt{\alpha} \times \frac{E_{Planck}}{k_B}$$

$$T_S = \sqrt{\alpha} \times T_{Planck}$$

$$T_{BH} = \frac{T_{Planck} \times m_{Planck}}{8\pi M}$$

$$T_{BH} = \frac{T_S \times m_S}{8\pi \times \alpha \times M}$$

Today's universe in Planck and Stoney units

Age	13.8×10^{9} years	$8.08 \times 10^{60}\ t_{Planck}$	$8.08 \times 10^{60} \frac{t_S}{\sqrt{\alpha}}$
Diameter	8.7×10^{26} m	$5.4 \times 10^{61}\ L_{Planck}$	$5.4 \times 10^{61} \frac{L_S}{\sqrt{\alpha}}$
Mass	3×10^{52} kg	approx. $10^{60}\ m_{Planck}$	approx. $10^{60} \frac{m_S}{\sqrt{\alpha}}$
Density	$9.9 \times 10^{-27}\ kg{\cdot}m^{-3}$	$1.8 \times 10^{-123} \frac{m_{Planck}}{L_{Planck}^3}$	$1.8 \times 10^{-123} \frac{\alpha\, m_S}{L_S^3}$
Temperature	2.725 K (Temperature of the cosmic microwave background radiation)	$1.9 \times 10^{-32}\ T_{Planck}$	$1.9 \times 10^{-32} \frac{T_S}{\sqrt{\alpha}}$
Cosmological constant	$1.1 \times 10^{-52}\ m^{-2}$	$2.9 \times 10^{-122} \frac{1}{L_{Planck}^2}$	$2.9 \times 10^{-122} \frac{\alpha}{L_S^2}$
Hubble constant	$2.2 \times 10^{-18}\ s^{-1}$	$1.18 \times 10^{-61} \frac{1}{t_{Planck}}$	$1.18 \times 10^{-61} \frac{\sqrt{\alpha}}{t_S}$

$$\text{Planck charge density} = \frac{\text{Planck charge}}{\text{Planck volume}} = \sqrt{\frac{c^{10} 4\pi\varepsilon_0}{\hbar^2 G^3}} = \frac{1}{t_{Planck}^2} \times \frac{1}{\sqrt{G \times \text{Coulomb constant}}}$$

$$\left\{ \textbf{Planck charge density} = \frac{\alpha}{t_S^2} \times \frac{1}{\sqrt{G \times \text{Coulomb constant}}} \right\}$$

$$\text{Planck energy density} = \frac{\text{Planck energy}}{\text{Planck volume}} = \frac{c^7}{G^2\hbar}$$

$$\textbf{Planck energy density} = \frac{m_{Planck}}{L_{Planck} \times t^2_{Planck}} = \frac{\alpha \times m_S}{L_S \times t_S^2}$$

$$\text{Planck force density} = \frac{\text{Planck force}}{\text{Planck volume}} = \frac{\hbar}{L^4_{Planck}\, t_{Planck}}$$

$$\text{Planck force density} = \frac{\alpha^{\frac{5}{2}}\, \hbar}{L_S^4\, t_S}$$

Hartree Energy:

$$E_h = 2\, R_\infty hc = \frac{\alpha hc}{2\pi a_0}$$

a_0 = Bohr radius

Hartree Force:

$$F_h = \frac{\text{Hartree Energy}}{\text{Bohr radius}}$$

$$F_h = \frac{\alpha hc}{2\pi a_0^2}$$

r_e = classical electron radius

$$F_h = \frac{\alpha^5 hc}{2\pi r_e^2}$$

$$F_h = \frac{\alpha hc}{2\pi a_0^2} = \frac{\alpha \hbar c}{a_0^2} = \frac{e^2}{4\pi\varepsilon_0 a_0^2}$$

- Z_0 = impedance of free space
- G_0 = conductance quantum

$$F_h = \frac{Z_0 G_0 hc}{8\pi a_0^2}$$

Hartree Momentum:

$$p_h = \frac{\hbar}{a_0} = \frac{\alpha^2 \hbar}{r_e}$$

R_K = von Klitzing constant

$$p_h = \frac{Z_0^2\, \hbar}{4R_K^2\, r_e}$$

Hartree Time:

$$t_h = \frac{\hbar}{E_h} = \frac{\hbar}{2R_\infty\, hc}$$

$$t_h = \frac{t_{Planck}}{4\pi R_\infty L_{Planck}} = \frac{t_S}{4\pi R_\infty L_S}$$

- $E_h \times t_h = \hbar$
- $p_h \times a_0 = \hbar$

$$E_h \times t_h = p_h \times a_0$$

$$E_h = p_h \times \frac{a_0}{t_h}$$

Hartree velocity:

$$\text{Planck speed} = \frac{\text{Hartree velocity}}{\text{Fine structure constant}}$$

$$v_h = \frac{a_0}{t_h} = \alpha \times c$$

$$v_h = \frac{\alpha\, L_{Planck}}{t_{Planck}} = \frac{\alpha\, L_S}{t_S}$$

$$E_{rest} = m_e c^2 = \frac{m_e v_h^2}{\alpha^2}$$

The threshold temperature below which the electron is effectively removed from the universe:

$$T_{threshold} = \frac{m_e c^2}{k_B} = \frac{\text{molar electron mass}}{\text{ideal gas constant}} \times \frac{v_h^2}{\alpha^2}$$

$$T_{threshold} = \frac{\text{molar electron mass}}{\text{ideal gas constant}} \times \frac{v_h^2 a_0}{r_e}$$

$$c_1 = 4\pi^2 \times \text{Planck angular momentum} \times (\text{Planck speed})^2$$

$$c_1 = 4\pi^2 \times \text{Planck angular momentum} \times \frac{(\text{Hartree velocity})^2}{(\text{Fine structure constant})^2}$$

$$c_1 = \frac{2\pi h v_h^2 a_0}{r_e}$$

$$c_2 = \frac{hc}{k_B} = \frac{\text{molar Planck constant}}{\text{ideal gas constant}} \times \frac{\text{Hartree velocity}}{\text{Fine structure constant}}$$

The Compton wavelength of the electron

$$\frac{h}{m_e c} = 2 \times \text{Quantum of circulation} \times \frac{\text{Fine structure constant}}{\text{Hartree velocity}}$$

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

$$v_h = \frac{\alpha}{\sqrt{\mu_0 \varepsilon_0}}$$

$$\Delta x\ \Delta p \geq \frac{\hbar}{2}$$

$$\Delta E\ \Delta t \geq \frac{\hbar}{2}$$

$$\Delta x\ \Delta p \geq \frac{\text{Bohr radius} \times \text{Hartree momentum}}{2}$$

$$\Delta E\ \Delta t \geq \frac{\text{Hartree energy} \times \text{Hartree time}}{2}$$

$$\frac{\Delta p}{\text{Hartree momentum}} \geq \frac{\text{Bohr radius}}{\Delta x}$$

$$\frac{\Delta E}{\text{Hartree energy}} \geq \frac{\text{Hartree time}}{\Delta t}$$

Sir Isaac Newton's famous Law of Universal Gravitation states that the force of gravitation is proportional to $\frac{1}{(\text{radius of the planet})^2}$ – which implies that if a radius of the planet shrinks by a factor of 2, then the force of gravitation at its surface must rise by a factor of 4.

$$\text{Planck force} = \frac{c^4}{G} = \frac{a_0^2 v_h^2}{G r_e^2}$$

$$\textbf{Planck power} = \frac{c^5}{G} = \frac{v_h^5}{G\alpha^5} = \frac{a_0^2 v_h^2}{G r_e^2} \times \frac{L_s}{t_s}$$

Black hole surface gravity is given by:

$$g_{BH} = \frac{c^4}{4GM}$$

$$\frac{\text{Planck force}}{4} = \text{Black hole mass} \times \text{Black hole surface gravity}$$

$$\frac{g_{BH}}{a_{Planck}} = \frac{m_{Planck}}{4M}$$

If $M = m_{Planck} = \sqrt{\frac{\hbar c}{G}}$:

$$g_{BH} = \frac{a_{Planck}}{4}$$

If $M = m_S = \sqrt{\frac{e^2}{4\pi\varepsilon_0 G}}$:

$$g_{BH} = \frac{a_{Planck}}{4\sqrt{\alpha}}$$

Lorentz factor:

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

A term by which relativistic mass, time and length changes for an object in motion

The Lorentz factor is always greater than 1 but it grows towards infinity as the object's velocity approaches the speed of light.

If v = **Hartree velocity**:

$$\gamma = \frac{1}{\sqrt{1-\alpha^2}}$$

- $m = \frac{m_0}{\sqrt{1-\alpha^2}}$
- $L = L_0\sqrt{1-\alpha^2}$
- $\Delta t = \frac{\Delta t_0}{\sqrt{1-\alpha^2}}$
- $KE = m_0c^2\left(\frac{1}{\sqrt{1-\alpha^2}} - 1\right)$

Fine structure constant $= \frac{1}{4 \times \text{magnetic coupling constant}}$

The wavelength of a relativistic particle is given by:

$$\lambda = \lambda_C\sqrt{\frac{c^2}{v^2} - 1}$$

If $v = \frac{c}{\sqrt{2}}$:

$$\lambda = \lambda_C$$

If v = $\mathbf{v_h}$:

$$\lambda = \lambda_C\sqrt{\frac{1}{\alpha^2} - 1} = \lambda_C\sqrt{\frac{a_0}{r_e} - 1}$$

$$\lambda = \lambda_C\sqrt{16\beta^2 - 1}$$

β ⟶ magnetic coupling constant

Hartree Temperature:

$$T_h = \frac{E_h}{k_B} = \frac{hc}{k_B} \times \frac{\alpha}{2\pi a_0}$$

$$T_h = \frac{c_2 \alpha}{2\pi a_0}$$

Hartree electric potential:

$$V_h = \frac{E_h}{e} = 4R\infty \times c \times \Phi_0 = \frac{4R_\infty \Phi_0}{\sqrt{\mu_0 \varepsilon_0}}$$

$$V_h = \frac{4R_\infty v_h \Phi_0}{\alpha}$$

$$V_h = \frac{E_h}{e} = \frac{hc}{e} \times \frac{\alpha}{2\pi a_0} = \frac{\Phi_0 v_h}{\pi a_0}$$

Hartree pressure:

$$P_h = \frac{E_h}{a_0^3} = \frac{\alpha hc}{2\pi a_0} \times \frac{1}{a_0^3}$$

$$P_h = v_h \times \frac{\hbar}{a_0^4}$$

Hartree current:

$$I_h = \frac{e}{\hbar} \times E_h = \frac{e}{\hbar} \times \frac{\alpha \hbar c}{a_0}$$

$$I_h = \frac{e \times v_h}{a_0}$$

Hartree charge density:

$$\frac{e}{a_0^3} = \frac{\alpha^{\frac{13}{2}} q_{Planck}}{r_e^3}$$

Hartree electric dipole moment:

$$e \times a_0 = \frac{e \times r_e}{\alpha^2}$$

Hartree electric dipole moment = $\alpha^{-\frac{3}{2}} \times q_{Planck} \times r_e$

The gravitational force between 2 electrons:

$$F_G = \frac{Gm_e^2}{r^2}$$

If F_G = **Hartree Force =** $\frac{\alpha\hbar c}{a_0^2}$**:**

$$\frac{\alpha\hbar c}{a_0^2} = \frac{Gm_e^2}{r^2}$$

$$r = \sqrt{\frac{\alpha_G}{\alpha}} \times a_0 = \sqrt{4 \times \beta \times \alpha_G} \times a_0$$

Distance between 2 electrons at which gravitational force between them is equal to Hartree force

The electrical force between 2 electrons:

$$F_E = \frac{e^2}{4\pi\varepsilon_0 r^2}$$

If F_G = **Hartree Force =** $\frac{\alpha\hbar c}{a_0^2}$:

$$\frac{\alpha\hbar c}{a_0^2} = \frac{e^2}{4\pi\varepsilon_0 r^2}$$

$$\boxed{r = a_0 = \frac{r_e}{\alpha^2}}$$

Distance between 2 electrons at which electrical force between them is equal to Hartree force

Quantum Chromodynamics Units:

QCD Length:

$$L_{QCD} = \frac{\hbar}{m_p c} = \text{reduced Compton wavelength of the proton}$$

QCD Time:

$$t_{QCD} = \frac{\hbar}{m_p c^2} = \frac{1}{\text{Compton angular frequency of the proton}}$$

QCD mass:

$$m_{QCD} = m_p = 1.673 \times 10^{-27}\ \text{kg}$$

QCD energy: $E_{QCD} = m_p c^2$

QCD Temperature:

$$T_{QCD} = \frac{E_{QCD}}{k_B} = \text{the threshold temperature below which the proton is effectively removed from the universe}$$

$$\Delta E \; \Delta t \geq \frac{\hbar}{2}$$

$$\Delta E \; \Delta t \geq \frac{E_{QCD} \times t_{QCD}}{2}$$

$$\frac{\Delta E}{E_{QCD}} \geq \frac{t_{QCD}}{\Delta t}$$

$$L_{QCD} \times t_{QCD} \times m_{QCD} = \frac{\hbar}{m_p c} \times \frac{\hbar}{m_p c^2} \times m_p$$

$$L_{QCD} \times t_{QCD} \times m_{QCD} = \frac{m_{Planck} \times L_{Planck} \times t_{Planck}}{\sqrt{\text{Proton gravitational coupling constant}}}$$

Astronomical range	Typical units
Distances to satellites	kilometers
Distances to near-Earth objects	lunar distance
Planetary distances	astronomical units, gigameters
Distances to nearby stars	parsecs, light-years
Distances at the galactic scale	kiloparsecs
Distances to nearby galaxies	megaparsecs

	Solar mass
Solar mass	1
Jupiter masses	1048
Earth masses	332950

$$F = eE$$

If F = **Hartree force** = $\frac{\alpha\hbar c}{a_0^2}$:

$$E = \frac{\Phi_0 v_h}{\pi a_0^2}$$

$$L_{QCD} \times T_{QCD} = \frac{\hbar}{m_p c} \times \frac{m_p c^2}{k_B}$$

$$L_{QCD} \times T_{QCD} = \frac{c_2}{2\pi}$$

$$\frac{L_{QCD}}{t_{QCD}} = c$$

$$\frac{L_{Planck}}{t_{Planck}} = c$$

$$\frac{L_S}{t_S} = c$$

$$\frac{L_{QCD}}{t_{QCD}} = \frac{L_{Planck}}{t_{Planck}} = \frac{L_S}{t_S}$$

$$c_1 = 2\pi h \times \frac{L_{QCD}}{t_{QCD}} \times \frac{L_{Planck}}{t_{Planck}}$$

$$E_{QCD} = m_{QCD} \times c^2$$

$$E_{QCD} = m_{QCD} \times \frac{L_{QCD}}{t_{QCD}} \times \frac{L_S}{t_S}$$

The electrical force between 2 protons is given by:

$$F_E = \frac{e^2}{4\pi\varepsilon_0 r^2}$$

If $r = L_{QCD}$:

$$F_E = \frac{e^2}{4\pi\varepsilon_0 L_{QCD}^2}$$

$$F_E = \textbf{Fine structure constant} \times \frac{E_{QCD}}{L_{QCD}}$$

The gravitational force between 2 protons is given by:

$$F_G = \frac{Gm_p^2}{r^2}$$

If r = L_{QCD}:

$$F_G = \frac{Gm_p^2}{L_{QCD}^2}$$

F_G = **Proton gravitational coupling constant** × $\frac{E_{QCD}}{L_{QCD}}$

The critical density of the universe:

$$\rho_{critical} = \frac{3H^2}{8\pi G}$$

If $\rho_{critical}$ = **Planck density** = $\frac{c^5}{\hbar G^2}$:

$$H = \sqrt{\frac{8\pi}{3 \times t_{Planck}}}$$

If the galaxy is taken to be spherical and the mass within the radius R is M, **the circular rotational velocity** at distance R is given by: $v_{rot} = \sqrt{\frac{GM}{r}}$**.** Thus, if v_{rot} is constant, it follows that M ∝ R, so that the total mass within radius R increases linearly with the distance from the centre.

$$\frac{m_e v^2}{2} = \frac{3k_B T}{2}$$

$$v^2 = 3 \times \frac{k_B}{m_e} \times T = 3 \times \frac{\text{ideal gas constant}}{\text{molar electron mass}} \times T$$

$$v^2 = \frac{3v_h^2}{\alpha^2\sqrt{\alpha_G}} \times \frac{T}{T_{Planck}}$$

- v_h = Hartree velocity and α_G = Electron gravitational coupling constant
- α = Fine structure constant and T_{Planck} = Planck temperature

$$\frac{m_e v^2}{2} = eV$$

$$v^2 = 2 \times \frac{e}{m_e} \times V = 2 \times \text{electron charge to mass ratio} \times V$$

$$v^2 = 2 \times \frac{\text{Faraday constant}}{\text{molar electron mass}} \times V$$

ε_g = Gravitoelectric gravitational constant

$$v^2 = 2V\sqrt{\frac{\alpha \times \varepsilon_0}{\alpha_G \times \varepsilon_g}}$$

Radiation density constant:

$$a = \frac{4\sigma}{c} = \frac{4\sigma \times t_{QCD}}{L_{QCD}}$$

If I = **Hartree current**:

$$\frac{e}{\hbar} \times E_h = \frac{dn_e}{dt} \times e$$

$$\frac{dn_e}{dt} = \frac{E_h}{\hbar}$$

$$\text{Rate of flow of electrons} = \frac{1}{\text{Hartree time}}$$

Extremophiles

↓

Organisms capable of living in extreme environments

Space debris

Artificial objects in space that are orbiting Earth but no longer serve a useful function

Precisely because Mars is an environment of great potential biological interest, it is possible that on Mars there are pathogens, organisms which, if transported to the terrestrial environment, might do enormous biological damage.

– Carl Sagan

The volume of the black hole:

$$V_{BH} = \frac{4\pi R_S^3}{3}$$

$$\frac{V_{BH}}{V_{Planck}} = \frac{32\pi}{3} \times \frac{M^3}{m_{Planck}^3}$$

If $M = m_{Planck}$:

$$V_{BH} = \frac{32\pi\, V_{Planck}}{3}$$

If $M = m_S = \sqrt{\alpha} \times m_{Planck}$:

$$V_{BH} = \frac{32\pi \times \alpha^{\frac{3}{2}} \times V_{Planck}}{3}$$

The surface area of the black hole:

$$A_{BH} = 4\pi R_S^2$$

$$\frac{A_{BH}}{A_{Planck}} = 16\pi \times \frac{M^2}{m_{Planck}^2}$$

If $M = m_{Planck}$:

$$A_{BH} = 16\pi \times A_{Planck}$$

If $M = m_S$:

$$A_{BH} = 16\pi \times \alpha \times A_{Planck}$$

$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$$

$$L_{QCD} = \frac{t_{QCD}}{\sqrt{\varepsilon_0 \mu_0}}$$

The Compton wavelength of the electron:

$$\lambda_{C,e} = 2\pi \times \alpha \times a_0$$

$$E_h \times \lambda_{C,e} = \frac{\alpha \hbar c}{a_0} \times (2\pi \times \alpha \times a_0)$$

$$E_h \times \lambda_{C,e} = \alpha^2 hc$$

$$E_h \times L_{QCD} = \frac{\alpha \hbar c}{a_0} \times \frac{\hbar}{m_p c}$$

$$E_h \times L_{QCD} = \alpha^2 \hbar c \times \frac{\text{electron mass}}{\text{proton mass}}$$

$$E_h \times L_{QCD} = \frac{\alpha^2 \hbar c}{1836.15267343}$$

Hartree energy:

$$E_h = \frac{\alpha\hbar c}{a_0} = \alpha\hbar c \times \frac{m_e c\alpha}{\hbar}$$

$$E_h = \alpha^2 m_e c^2$$

$$\alpha = \sqrt{\frac{E_h}{m_e c^2}} = \frac{e^2}{q^2_{Planck}} = \frac{Z_0 G_0}{4} = \sqrt{\frac{T_h}{T_{threshold}}}$$

$$E_h \times L_{Planck} = \alpha^2 m_e c^2 \times \sqrt{\frac{\hbar G}{c^3}}$$

α_G = electron gravitational coupling constant

$$E_h \times L_{Planck} = \alpha^2 \sqrt{\alpha_G}\, \hbar c$$

$$E_h \times L_S = \alpha^2 m_e c^2 \times (\sqrt{\alpha}\, L_{Planck})$$

$$E_h \times L_S = \alpha^{\frac{5}{2}} \sqrt{\alpha_G}\, \hbar c$$

Hartree energy × Rydberg wavelength = 2hc

$$E_h \times r_S = \alpha^2 m_e c^2 \times \frac{2Gm_e}{c^2}$$

$$E_h \times r_S = 2\alpha^2 \times \alpha_G \times \hbar c$$

$$F_G = \frac{Gm_p m_e}{r^2}$$

$$F_G = \frac{Gm_{Planck}^2}{r^2}\sqrt{\text{Proton gravitational coupling constant}} \times \sqrt{\text{Electron gravitational coupling constant}}$$

$$F_G = \frac{\hbar c}{r^2}\sqrt{\text{Proton gravitational coupling constant}} \times \sqrt{\text{Electron gravitational coupling constant}}$$

- $E_h \times t_{QCD} = \dfrac{\alpha^2 \hbar}{1836.15267343}$
- $E_h \times t_{Planck} = \alpha^2 \sqrt{\alpha_G}\, \hbar$
- $E_h \times t_S = = \alpha^{\frac{5}{2}} \sqrt{\alpha_G}\, \hbar$

"there are no arbitrary constants ... nature is so constituted that it is possible logically to lay down such strongly determined laws that within these laws only rationally determined constants occur (not constants, therefore, whose numerical value could be changed without destroying the theory)."

— **Albert Einstein**

$$\Delta\lambda = \frac{h}{m_e c}(1 - \cos\theta)$$

If $\Delta\lambda = L_{QCD}$:

$$L_{QCD} = \frac{h}{m_e c}(1 - \cos\theta)$$

$$\theta = \cos^{-1}\left(1 - \frac{m_e}{2\pi m_p}\right)$$

The wavelength shift of the scattered photon in an angle of $\theta = \cos^{-1}\left(1 - \frac{m_e}{2\pi m_p}\right)$ is equal to the QCD length.

$$I_h \times t_{QCD} = \frac{e}{\hbar} \times E_h \times \frac{\hbar}{m_p c^2}$$

$$I_h \times t_{QCD} = e \times \alpha^2 \times \frac{m_e}{m_p} = \frac{e \times \alpha^2}{1836.15267343}$$

$$I_h \times t_{QCD} = e \times \alpha^2 \times \sqrt{\frac{\text{electron gravitational coupling constant}}{\text{proton gravitational coupling constant}}}$$

$$I_h \times t_{Planck} = \frac{e}{\hbar} \times E_h \times \frac{\hbar}{m_{Planck}c^2}$$

↓

$$\left\{ I_h \times t_{Planck} = e \times \alpha^2 \times \sqrt{\text{electron gravitational coupling constant}} \right\}$$

$$I_h \times t_S = \frac{e}{\hbar} \times E_h \times (\sqrt{\alpha} \times t_{Planck})$$

$$I_h \times t_S = e \times \alpha^{\frac{5}{2}} \times \sqrt{\text{electron gravitational coupling constant}}$$

$$\Phi_0 \times I_h = \frac{h}{2e} \times \frac{e}{\hbar} \times E_h$$

↓

$$\left\{ \Phi_0 \times I_h = \pi\, E_h \right\}$$

$$\frac{\text{Hartree energy}}{\text{Planck energy}} = (\text{Fine structure constant})^2 \times \sqrt{\text{electron gravitational coupling constant}}$$

$$\frac{\text{Hartree force}}{\text{Planck force}} = \alpha^2 \times \sqrt{\text{electron gravitational coupling constant}} \times \frac{\text{Planck length}}{\text{Bohr radius}}$$

Hartree force:

$$F_h = \frac{\alpha^2 m_e c^2}{a_0} = 2\pi\alpha^3 \frac{m_e c^2}{\lambda_{C,e}}$$

$$\left\{ \text{Rest mass energy of electron} = \frac{1}{2\pi\alpha^3} \times \text{Hartree force} \times \text{Compton wavelength of electron} \right\}$$

$$F_h = = 2\pi\alpha^3 \frac{m_e^2 c^3}{h} = 2\pi\alpha^3 \frac{k_B T_{threshold}^2}{c_2}$$

$$E_{QCD} \times t_{Planck} = m_p c^2 \times \frac{\hbar}{m_{Planck} c^2}$$

$$E_{QCD} \times t_{Planck} = \sqrt{\text{proton gravitational coupling constant}} \times \hbar$$

$$E_{QCD} \times t_S = m_p c^2 \times \left(\sqrt{\alpha} \times \frac{\hbar}{m_{Planck} c^2}\right)$$

↓

$$E_{QCD} \times t_S = \sqrt{\text{Fine structure constant} \times \text{proton gravitational coupling constant}} \times \hbar$$

$$E_{QCD} \times t_h = m_p c^2 \times \frac{\hbar}{E_h}$$

$$E_{QCD} \times t_h = \frac{1836.15267343 \times \hbar}{\alpha^2}$$

Einstein's Photoelectric Equation:

The energy required to eject the electron from the metal surface

$$E = W_0 + KE_{electron}$$

$$\frac{m_e v^2}{2} = h\,(\upsilon - \upsilon_0)$$

$$v = 2\sqrt{Q_0(\upsilon - \upsilon_0)}$$

$$eV_S = h\,(\upsilon - \upsilon_0)$$

$$V_S = 2\,\Phi_0\,(\upsilon - \upsilon_0)$$

$$a_{Planck} \times t_h = \frac{c}{t_{Planck}} \times \frac{\hbar}{\alpha^2 m_e c^2}$$

$$a_{Planck} \times t_h = \frac{c}{\alpha^2\sqrt{\text{electron gravitational coupling constant}}}$$

$$a_{Planck} \times t_{QCD} = \frac{c}{t_{Planck}} \times \frac{\hbar}{m_p c^2}$$

$$\downarrow$$

$$\left\{ a_{Planck} \times t_{QCD} = \frac{c}{\sqrt{\text{proton gravitational coupling constant}}} \right\}$$

$$E_h \times \mu_B = \alpha^2 m_e c^2 \times \frac{e\hbar}{2m_e}$$

$$\downarrow$$

$$\left\{ E_h \times \mu_B = \frac{\alpha^2 \times e \times c_1}{8\pi^2} \right\}$$

$$E_h \times \mu_N = \alpha^2 m_e c^2 \times \frac{e\hbar}{2m_p}$$

$$E_h \times \mu_N = \frac{\alpha^2 \times e \times c_1}{8\pi^2} \times \frac{m_e}{m_p} = \frac{\alpha^2 \times e \times c_1}{14689.2213874\ \pi^2}$$

$$E_{Planck} \times \mu_B = m_{Planck}c^2 \times \frac{e\hbar}{2m_e}$$

$$E_{Planck} \times \mu_B = \frac{e \times c_1}{\sqrt{\text{electron gravitational coupling constant}} \times 8\pi^2}$$

$$E_S \times \mu_B = \sqrt{\alpha}\, m_{Planck}c^2 \times \frac{e\hbar}{2m_e}$$

$$E_S \times \mu_B = \frac{\sqrt{\text{Fine structure constant}} \times e \times c_1}{\sqrt{\text{electron gravitational coupling constant}} \times 8\pi^2}$$

$$\text{Planck intensity} = \frac{\text{Planck power}}{\text{Planck area}} = \frac{c^8}{\hbar G^2} = \frac{m_S c^2}{t_S} \times \frac{\alpha}{L_S^2} = \frac{\alpha m_S}{t_S^3}$$

$$\text{Planck intensity} = \frac{(\text{Planck force})^2}{\hbar} = \frac{4\pi^2 \times (\text{Planck power})^2}{\text{First radiation constant}}$$

$$\textbf{Planck power} = \frac{1}{2\pi}\sqrt{\text{First radiation constant} \times \text{Planck intensity}}$$

$$\text{Planck Intensity} = \frac{m_e^2 c^4}{\hbar} \times \frac{c^4}{G^2 m_e^2} = \frac{4\hbar\omega_C^2}{r_S}$$

- ω_C = Compton angular frequency of the electron
- r_S = Schwarzschild radius of the electron

$$E_{QCD} \times \mu_B = m_p c^2 \times \frac{e\hbar}{2m_e}$$

↓

$$\left\{ E_{QCD} \times \mu_B = \frac{\sqrt{\text{proton gravitational coupling constant}} \times e \times c_1}{\sqrt{\text{electron gravitational coupling constant}} \times 8\pi^2} \right\}$$

$$E_{QCD} \times \mu_N = m_p c^2 \times \frac{e\hbar}{2m_p}$$

$$E_{QCD} \times \mu_N = \frac{e \times c_1}{8\pi^2}$$

$$E_{Planck} \times \mu_N = m_{Planck} c^2 \times \frac{e\hbar}{2m_p}$$

$$E_{Planck} \times \mu_N = \frac{e \times c_1}{\sqrt{\text{proton gravitational coupling constant}} \times 8\pi^2}$$

$$E_S \times \mu_N = \sqrt{\alpha}\, m_{Planck} c^2 \times \frac{e\hbar}{2m_p}$$

$$E_S \times \mu_N = \frac{\sqrt{\text{Fine structure constant}} \times e \times c_1}{\sqrt{\text{proton gravitational coupling constant}} \times 8\pi^2}$$

Planck Temperature:

$$T_{Planck} = \sqrt{\frac{\hbar c^5}{G k_B^2}}$$

$$T_{Planck} = \sqrt{\frac{c^2}{G m_e} \times \frac{hc}{2\pi k_B}} \times \sqrt{\frac{m_e c^2}{k_B}}$$

$$T_{Planck} = \sqrt{\frac{c_2 \times T_{threshold}}{\pi r_S}}$$

"**Fine Structure Constant:** Fundamental numerical constant of atomic physics and quantum electrodynamics, defined as the square of the charge of the electron divided by the product of Planck's constant and the speed of light."

— **Steven Weinberg**

$$F_h \times t_h = \frac{E_h}{a_0} \times \frac{\hbar}{E_h}$$

$$F_h \times t_h = \alpha\, m_e c$$

$$F_h \times t_h = \text{Fine structure constant} \times \sqrt{\text{electron gravitational coupling constant}} \times \text{Planck momentum}$$

$$F_h \times t_{Planck} = \frac{E_h}{a_0} \times \frac{\hbar}{E_{Planck}}$$

$$F_h \times t_{Planck} = \alpha^3 \times \text{electron gravitational coupling constant} \times \text{Planck momentum}$$

$$F_h \times t_S = \frac{E_h}{a_0} \times \frac{\sqrt{\alpha} \times \hbar}{E_{Planck}}$$

$$F_h \times t_S = \alpha^{\frac{7}{2}} \times \text{electron gravitational coupling constant} \times \text{Planck momentum}$$

Reduced mass of hydrogen atom:

$$\mu = \frac{m_e m_p}{(m_e + m_p)}$$

- $\mu \leq m_e$
- $\mu \leq m_p$

$$\mu = \frac{\sqrt{\text{electron gravitational coupling constant}} \times \sqrt{\text{proton gravitational coupling constant}} \times \text{Planck mass}}{(\sqrt{\text{electron gravitational coupling constant}} + \sqrt{\text{proton gravitational coupling constant}})}$$

Hartree Power:

$$P_h = F_h \times v_h = \frac{\alpha^2 m_e c^2}{a_0} \times \alpha c$$

$$P_h = \frac{\alpha^4 m_e^2 c^4}{\hbar}$$

$P_h = \alpha^4 \times$ Electron gravitational coupling constant $\times$ Planck power

$P_h \times t_{Planck} = \alpha^4 \times$ Electron gravitational coupling constant $\times$ Planck energy

$P_h \times t_S = \alpha^{\frac{9}{2}} \times$ Electron gravitational coupling constant $\times$ Planck energy

"The fine-structure constant derives its name from its origin. It first appeared in Sommerfeld's work to explain the fine details of the hydrogen spectrum. ... Since Sommerfeld expressed the energy states of the hydrogen atom in terms of the constant [alpha], it came to be called the fine-structure constant."

— **John S. Rigden**

$$a_{Planck} \times Q_0 = \frac{c}{t_{Planck}} \times \frac{h}{2m_e}$$

$$a_{Planck} \times Q_0 = \frac{\pi \times c^3}{\sqrt{\text{electron gravitational coupling constant}}}$$

$$a_{Planck} \times \Phi_0 = \frac{c}{t_{Planck}} \times \frac{h}{2e}$$

$$a_{Planck} \times \Phi_0 = \frac{\pi c \times \text{Planck voltage}}{\sqrt{\text{Fine structure constant}}}$$

$$E_h \times Q_0 = \alpha^2 m_e c^2 \times \frac{h}{2m_e}$$

↓

$$\left\{ E_h \times Q_0 = \frac{\alpha^2 c_1}{4\pi^2} \right\}$$

$$E_{QCD} \times Q_0 = m_p c^2 \times \frac{h}{2m_e}$$

$$E_{QCD} \times Q_0 = \frac{\sqrt{\text{proton gravitational coupling constant}} \times c_1}{4\pi^2 \sqrt{\text{electron gravitational coupling constant}}}$$

$$E_{Planck} \times Q_0 = m_{Planck}c^2 \times \frac{h}{2m_e}$$

$$E_{Planck} \times Q_0 = \frac{c_1}{4\pi^2\sqrt{\text{electron gravitational coupling constant}}}$$

$$E_S \times Q_0 = \sqrt{\alpha}\, m_{Planck}c^2 \times \frac{h}{2m_e}$$

$$\downarrow$$

$$E_S \times Q_0 = \frac{\sqrt{\text{Fine structure constant}} \times c_1}{4\pi^2\sqrt{\text{electron gravitational coupling constant}}}$$

A quantum fluctuation can create an proton antiproton pair with energy $\Delta E \geq 2m_pc^2$ provided the fluctuation lives less than the time $\Delta t \leq \frac{\hbar}{\Delta E}$. In that time, the proton and antiproton can separate by a distance of order $\Delta x = c \times \Delta t$. As they separate they gain energy $eE \times \Delta x$, in the electric field with strength E. If they gain sufficient energy to compensate for their rest mass, they no longer have to annihilate: they can become real particles. The condition for real proton− antiproton pair creation is therefore that the electric field be greater than a critical value, $E_{critical}$ given by:

$$e\, E_{critical} \times \left(c \times \frac{\hbar}{2m_pc^2}\right) = 2m_pc^2$$

$$E_{critical} = \frac{4m_p^2c^3}{\hbar e}$$

A modern mathematical proof is not very different from a modern machine, or a modern test setup: the simple fundamental principles are hidden and almost invisible under a mass of technical details.

— Hermann Weyl

$$F_h \times t_{QCD} = \frac{E_h}{a_0} \times \frac{\hbar}{m_p c^2}$$

$$F_h \times t_{QCD} = \frac{\alpha^3 \times \text{electron gravitational coupling constant} \times \text{Planck momentum}}{\sqrt{\text{proton gravitational coupling constant}}}$$

$$P_h \times t_{QCD} = \frac{\alpha^4 m_e^2 c^4}{\hbar} \times \frac{\hbar}{m_p c^2}$$

$$P_h \times t_{QCD} = \frac{\alpha^4 \times \text{Electron gravitational coupling constant} \times \text{Planck energy}}{\sqrt{\text{Proton gravitational coupling constant}}}$$

Number of electron charges that make up one Planck charge:

$$n = \frac{\text{Planck charge}}{\text{Electron charge}} = \frac{1}{\sqrt{\alpha}} = \frac{2}{\sqrt{\text{impedance of free space} \times \text{conductance quantum}}}$$

The radius of photon orbit:

$$r = \frac{3GM}{c^2}$$

Any photon orbiting below this distance will plunge into the black hole, while photon that remains further away will spiral out towards infinity.

If $M = m_{Planck} = \sqrt{\frac{\hbar c}{G}}$:

r = 3 × Planck length

The electric potential energy between 2 electrons:

$$E_p = \frac{e^2}{4\pi\varepsilon_0 r}$$

If E_p = **Hartree energy**:

$$\alpha^2 m_e c^2 = \frac{e^2}{4\pi\varepsilon_0 r}$$

$$r = \frac{r_e}{\alpha^2}$$

Distance between 2 electrons at which the electric potential energy between them is equal to **Hartree energy**

The gravitational potential energy between 2 electrons:

$$E_p = \frac{Gm_e^2}{r}$$

If E_p = **Hartree energy**:

$$\alpha^2 m_e c^2 = \frac{Gm_e^2}{r}$$

$$r = \frac{r_S}{2 \times \alpha^2}$$

Distance between 2 electrons at which the gravitational potential energy between them is equal to **Hartree energy**

"It doesn't matter how beautiful your theory is, it doesn't matter how smart you are. If it doesn't agree with experiment, it's wrong. "

− Richard P. Feynman

If $\frac{e^2}{4\pi\varepsilon_0 r}$ = **Planck energy** = $m_{Planck}c^2$:

$$r = \sqrt{\text{electron gravitational coupling constant}} \times r_e$$

Distance between 2 electrons at which the electric potential energy between them is equal to **Planck energy**

If $\frac{e^2}{4\pi\varepsilon_0 r}$ = **Stoney energy** = $\sqrt{\alpha}\, m_{Planck}c^2$:

$$r = \sqrt{\frac{\text{electron gravitational coupling constant}}{\text{Fine structure constant}}} \times r_e$$

Distance between 2 electrons at which the electric potential energy between them is equal to **Stoney energy**

If $\frac{Gm_e^2}{r}$ = **Planck energy** = $m_{Planck}c^2$:

$$r = \sqrt{\text{electron gravitational coupling constant}} \times \frac{r_S}{2}$$

Distance between 2 electrons at which the gravitational potential energy between them is equal to **Planck energy**

If $\frac{Gm_e^2}{r}$ = **Stoney energy** = $\sqrt{\alpha}\, m_{Planck}c^2$:

$$r = \sqrt{\frac{\text{electron gravitational coupling constant}}{\text{Fine structure constant}}} \times \frac{r_S}{2}$$

Distance between 2 electrons at which the gravitational potential energy between them is equal to **Stoney energy**

"Primitive life is very common and intelligent life is fairly rare. Some would say it has yet to occur on Earth."

– Stephen Hawking

$$F_G = \frac{Gm_1 m_2}{r^2}$$

Because $r_S = \frac{2Gm}{c^2}$:

$$F_G = \frac{F_{Planck}}{4} \times \frac{r_{S_1} \times r_{S_2}}{r^2}$$

$\frac{F_{Planck}}{4}$ → Proportionality constant

$$F_G \propto \frac{r_{S_1} \times r_{S_2}}{r^2}$$

Niels Bohr was a Danish physicist who is generally regarded as one of the foremost physicists of the 20th century. He was the first to apply the quantum concept, which restricts the energy of a system to certain discrete values, to the problem of atomic and molecular structure. For that work he received the Nobel Prize for Physics in 1922. His manifold roles in the origins and development of quantum physics may be his most-important contribution, but through his long career his involvements were substantially broader, both inside and outside the world of physics.

In 1911, fresh from completion of his PhD, the young Danish physicist **Niels Bohr** left Denmark on a foreign scholarship headed for the Cavendish Laboratory in Cambridge to work under J. J. Thomson on the structure of atomic systems. At the time, Bohr began to put forth the idea that since light could no long be treated as continuously propagating waves, but instead as **discrete energy packets** (as articulated by Max Planck and Albert Einstein), why should the classical Newtonian mechanics on which **Thomson's model** was based hold true? It seemed to Bohr that the **atomic model** should be modified in a similar way. If electromagnetic energy is quantized, i.e. restricted to take on only integer values of hυ, where υ is the frequency of light, then it seemed reasonable that the mechanical energy associated with the energy of atomic electrons is also quantized. However, Bohr's still somewhat vague ideas were not well received by Thomson, and Bohr decided to move from Cambridge after his first year to a place where his concepts about **quantization of electronic motion** in atoms would meet less opposition. He chose the University of Manchester, where the chair of physics was held by **Ernest Rutherford**. While in Manchester, Bohr learned about the nuclear model of the atom proposed by Rutherford. To overcome the difficulty associated with the classical collapse of the electron into the nucleus, Bohr proposed that the orbiting electron could only exist in certain special states of motion - called stationary states, in which no electromagnetic radiation was emitted. In these states, the angular momentum of the electron L takes on integer values of Planck's constant divided by 2π, denoted by $\hbar = \frac{h}{2\pi}$ (pronounced h-bar). In these stationary states, the electron angular momentum can take on values $\hbar$, $2\hbar$, $3\hbar$... but never non-integer values. This is known as quantization of angular momentum, and was one of **Bohr's key hypotheses**. He imagined the atom as consisting of electron waves of wavelength $\lambda = \frac{h}{m_e v} = \frac{h}{p}$ endlessly circling atomic nuclei. In his picture, only orbits with circumferences corresponding to an integral multiple of electron wavelengths could survive without **destructive interference** (i.e., $\mathbf{r} = \frac{n\hbar}{m_e v}$ could survive without destructive interference). For circular orbits, the position vector of the electron **r** is always perpendicular to its linear momentum p. The angular momentum **L** has magnitude $\mathbf{m_e vr}$ in this case. Thus Bohr's postulate of quantized angular momentum is equivalent to $\mathbf{m_e vr} = n\hbar$ where **n** is a positive integer called principal quantum number. It tells us what energy level the electron occupies.

For an electron moving in a circular orbit of radius r:

$$\omega = \frac{v}{r}$$

Since $\lambda = \frac{h}{m_e v} = \frac{h}{p}$ (de Broglie relation),

$$pv_p = \frac{hv_p}{\lambda} = h\upsilon = \hbar\omega$$

where $\hbar = \frac{h}{2\pi}$ is the reduced Planck constant, $\omega = 2\pi\upsilon$ is the angular frequency and v_p is the phase velocity.

$$pv_p = \frac{\hbar v}{r}$$

Since $n\hbar = pr$ (quantization of angular momentum),

$$v = n \times v_p$$

The velocity of the electron or the group velocity of the corresponding matter wave associated with the electron is the integral multiple of the **phase velocity** of the corresponding matter wave associated with the electron.

Quantum of circulation: $Q_0 = \frac{h}{2m_e}$

$\mathbf{m_e v r} = n\hbar$ →

$$v = \frac{nQ_0}{\pi r} \longrightarrow v = \frac{2Q_0}{\lambda}$$

$$\omega = \frac{v}{r} = \frac{nQ_0}{\pi r^2} = \frac{nQ_0}{\text{Area of circular orbit}}$$

By the de Broglie hypothesis, we see that:

$$\frac{pv_p}{\lambda} = \frac{h\upsilon}{\lambda}$$

$$\frac{pv}{n\lambda} = \frac{h\upsilon}{\lambda}$$

Substituting $n\lambda = 2\pi r$,

$$\frac{m_e v^2}{r} = 2\pi\frac{h\upsilon}{\lambda}$$

The classical description of the nuclear atom is based upon the Coulomb attraction between the positively charged nucleus and the negative electrons orbiting the nucleus. Furthermore, we consider only circular orbits. The electron, with mass $\mathbf{m_e}$ and charge e^- moves in a circular orbit of radius **r** with constant velocity **v**. The attractive **Coulomb force** provides the necessary acceleration to maintain orbital motion. (Note we neglect the motion of the nucleus since its mass is much greater than the electron). The total force on the electron is thus

$$F = \frac{Ze^2}{4\pi\varepsilon_0 r^2} = \frac{m_e v^2}{r}$$

where $\varepsilon_0 = 8.854 \times 10^{-12} \frac{F}{m}$ is the permittivity of free space.

$$F = 2\pi \frac{h\upsilon}{\lambda}$$

$$-\frac{Ze^2}{4\pi\varepsilon_0 r} = -2\pi r \frac{h\upsilon}{\lambda}$$

Substituting $2\pi r = n\lambda$,

$$-\frac{Ze^2}{4\pi\varepsilon_0 r} = U = -nh\upsilon$$

The potential energy of the electron

The negative sign indicates that it requires energy to pull the orbiting electron away from the nucleus.

From the equation:

$$KE = \frac{m_e v^2}{2} = \frac{pv}{2}$$

we can determine the kinetic energy of the electron (neglecting relativistic effects)

Substituting $p = \frac{n\hbar}{r}$,

$$KE = \frac{n\hbar v}{2r} = \frac{n\hbar\omega}{2} = \frac{nh\upsilon}{2}$$

The kinetic energy of the electron is the integral multiple of $\frac{h\upsilon}{2}$

The total energy of the electron E = KE + U is thus:

$$E = KE + U = \frac{nh\upsilon}{2} + (-nh\upsilon)$$

$$E = -\frac{nh\upsilon}{2}$$

The frequency of photon absorbed or emitted when transition occurs between two stationary states that differ in energy by ΔE, is given by:

$$\upsilon_{photon} = \frac{\Delta E}{h} = \frac{E_2 - E_1}{h}$$

where E_1 and E_2 denote the energies of the lower and higher allowed energy states respectively. This expression is commonly known as **Bohr's frequency rule**.

$$\upsilon_{photon} = \frac{\left(-\frac{n_2 h\upsilon_2}{2}\right) - \left(-\frac{n_1 h\upsilon_1}{2}\right)}{h}$$

$$n_1\upsilon_1 - n_2\upsilon_2 = 2\upsilon_{photon}$$

In physics (specifically, celestial mechanics), escape velocity is the minimum speed needed for an electron to escape from the electrostatic influence of a nucleus. If the **kinetic energy** $\frac{m_e v^2}{2}$ of the electron is equal in magnitude to the **potential energy** $\frac{Ze^2}{4\pi\varepsilon_0 r}$, then electron could escape from the electrostatic field of a nucleus.

$$\frac{m_e v^2}{2} = \frac{Ze^2}{4\pi\varepsilon_0 r}$$

$$\frac{m_e v^2}{2} = nh\upsilon$$

$$v = v_{escape} = \sqrt{\frac{2nh\upsilon}{m_e}} = \sqrt{4nQ_0\upsilon}$$

Orbital velocity:

$$\frac{Ze^2}{4\pi\varepsilon_0 r^2} = \frac{m_e v^2}{r}$$

$$v_{orbital} = \sqrt{\frac{Ze^2}{4\pi\varepsilon_0 r m_e}} = \sqrt{\frac{nh\upsilon}{m_e}}$$

$$v_{orbital} = c\sqrt{\frac{Z \times \text{classical electron radius}}{r}} = c \times \text{Fine structure constant}\sqrt{\frac{Z \times \text{Bohr radius}}{r}}$$

$$v_{orbital} = \textbf{Hartree velocity}\sqrt{\frac{Z \times \text{Bohr radius}}{r}}$$

Total energy of the electron:

$$E = -\frac{nh\upsilon}{2}$$

$$\frac{E}{m_e c^2} = -\frac{n\upsilon}{2\upsilon_C}$$

$\upsilon_C = \frac{m_e c^2}{h}$ is the Compton frequency of the electron.

$$\frac{Ze^2}{4\pi\varepsilon_0 r^2} = \frac{m_e v^2}{r}$$

$$\frac{Ze^2}{4\pi\varepsilon_0 r^2} = m_e\omega^2 r$$

$$\frac{Ze^2}{4\pi\varepsilon_0 r^2} = m_e \times \frac{4\pi^2}{T^2} \times r$$

$$\frac{4\pi^2}{T^2} = r_e\, c^2 \times \frac{Z}{r^3}$$

where r_e denote the Classical electron radius

$$\left\{ T^2 \propto \frac{r^3}{Z} \right\}$$

$2\pi r = n\lambda$

(standing-wave condition)

+

$$\lambda = \frac{h}{p}$$

(de Broglie relation)

↓

$L = n\hbar$

(Bohr's postulate)

"The very nature of the quantum theory ... forces us to regard the space-time coordination and the claim of causality, the union of which characterizes the classical theories, as complementary but exclusive features of the description, symbolizing the idealization of observation and description, respectively."

— Niels Bohr

The moment of inertia of an electron in n^{th} orbit is:

$$I = n \times m_e r^2$$

particle ← $\lambda = \frac{h}{m_e v}$ → wave

Planetary Model failed to explain stability of atoms in accordance with classical laws of physics

$$m_e r = \frac{n\hbar}{v}$$

$$I = n^2 \times \frac{\hbar r}{v} = \frac{n^2 \hbar}{\omega}$$

$$n = \sqrt{\frac{I\omega}{\hbar}}$$

The acceleration of the electron:

$$a = \frac{v^2}{r} = \omega \times v$$

$$a = \frac{2\pi}{T}\sqrt{\frac{nh\upsilon}{m_e}} = \frac{2\pi\sqrt{2nQ_0\upsilon}}{T}$$

Plum Pudding Model failed to explain large-angle deflections of scattered alpha particles

$$F = Z \times \frac{e^2}{4\pi\varepsilon_0} \times \frac{1}{r^2}$$

$$F = Z \times \text{Fine structure constant} \times \frac{hc}{2 \times \text{Area of the circular orbit}}$$

$$\frac{Ze^2}{4\pi\varepsilon_0 r^2} = \frac{m_e}{r} \times \frac{n^2\hbar^2}{m_e^2 r^2}$$

$$r = \frac{n^2}{Z} \times \text{Bohr radius} = \frac{n^2 r_e}{Z\alpha^2}$$

Rydberg formula:

$$\upsilon_{photon} = \textbf{Rydberg frequency} \times Z^2 \frac{n_2^2 - n_1^2}{n_1^2 n_2^2}$$

For hydrogen atom: Z = 1

$$\upsilon_{photon} = \textbf{Rydberg frequency} \times \frac{n_2^2 - n_1^2}{n_1^2 n_2^2}$$

$$\frac{n_1\upsilon_1 - n_2\upsilon_2}{2} = \textbf{Rydberg frequency} \times \frac{n_2^2 - n_1^2}{n_1^2 n_2^2}$$

$$\textbf{Rydberg frequency} = \frac{n_1^2 n_2^2 (n_1\upsilon_1 - n_2\upsilon_2)}{2(n_2^2 - n_1^2)}$$

n_1	n_2	Series Name
1	$2-\infty$	Lyman
2	$3-\infty$	Balmer
3	$4-\infty$	Paschen
4	$5-\infty$	Brackett
5	$6-\infty$	Pfund
6	$7-\infty$	Humphreys

- **Area of ellipse (integral form):**

$$\oint L d\varphi$$

- **Area of ellipse (geometrical form):**

$$2\pi n\hbar$$

Bohr-Sommerfeld quantization rule for angular momentum:

$$\oint L d\varphi = 2\pi n\hbar$$

Bohr quantization rule

In the case of circular orbits: L is constant and

$$\oint L d\varphi = L\int_0^{2\pi} d\varphi = 2\pi n\hbar \rightarrow L = n\hbar$$

Total energy of the electron:

$$E = KE + U = -\frac{hc\,R_\infty}{n^2} = -\frac{nh\upsilon}{2}$$

$$\left\{ R_\infty = \frac{n^3\upsilon}{2c} \right\}$$

$$\frac{\text{Hartree electric potential}}{\text{Planck voltage}} = \frac{E_h}{e} \times \frac{q_{Planck}}{E_{Planck}}$$

$$\alpha^{\frac{3}{2}} \times \sqrt{\text{electron gravitational coupling constant}}$$

Ionization energy

↓

Electron charge × Ionization potential

{ The minimum energy required to liberate the electron from the binding of nucleus. }

$$\frac{m_e v^2}{r} = \frac{Ze^2}{4\pi\varepsilon_0 r^2}$$

↓

$$v = \frac{Ze^2}{4\pi\varepsilon_0 n\hbar} = \frac{Z\alpha c}{n} = \sqrt{\frac{nh\upsilon}{m_e}}$$

$$\alpha = \frac{1}{Z}\sqrt{\frac{n^3\upsilon}{\upsilon_C}}$$

Separation energy

↓

The energy needed to remove a proton or a neutron from an atomic nucleus.

Ground state → Excited state

First excitation potential $= E_2 - E_1$

$$-\frac{hc\,R_\infty}{n_2^2} + \frac{hc\,R_\infty}{n_1^2} = -3.4 + 13.6 = 10.2 \text{ eV}$$

Second excitation potential $= E_3 - E_1$

$$-\frac{hc\,R_\infty}{n_3^2} + \frac{hc\,R_\infty}{n_1^2} = -1.5 + 13.6 = 12.1 \text{ eV}$$

Rydberg formula for the spectrum of the hydrogen atom:

$$\lambda_{max} = \frac{n_1^2 n_2^2}{(n_2^2 - n_1^2)R_\infty}$$

$$\lambda_{min} = \frac{n_1^2}{R_\infty}$$

$$\left\{ \frac{\lambda_{max}}{\lambda_{min}} = \frac{n_2^2}{(n_2^2 - n_1^2)} \right\}$$

E= hυ

Because $E = mc^2$:

The Planck constant relates mass to frequency.

Bohr's model does not work for systems with more than one electron.

$$e = \frac{2}{K_J\ R_K}$$

Fine structure constant:

$$\alpha = \frac{e^2}{2\varepsilon_0 ch} = \frac{h\ K_J^2}{8\varepsilon_0 c} = \frac{h\ K_J^2}{8}\sqrt{\frac{\mu_0}{\varepsilon_0}}$$

$$K_J = \frac{1}{\text{Magnetic flux quantum}} = \text{Josephson constant}$$

$$h = \frac{4}{K_J^2\ R_K}$$

R_K = von Klitzing constant

Nothing can better illustrate the positive and hectic pace of progress which the art of experimenters has made over the past twenty years, than the fact that since that time, not only one, but a great number of methods have been discovered for measuring the mass of a molecule with practically the same accuracy as that attained for a planet.

– Max Planck

$$\Delta\alpha = \alpha_{previous} - \alpha_{now}$$

If the **fine-structure constant** really is a constant, then any experiment should show that

$$\Delta\alpha = 0$$

Any value far away from zero would indicate that α does change over time. So far, most experimental data is consistent with α being constant.

Even if there is only one possible unified theory, it is just a set of rules and equations. What is it that breathes fire into the equations and makes a universe for them to describe? The usual approach of science of constructing a mathematical model cannot answer the questions of why there should be a universe for the model to describe. Why does the universe go to all the bother of existing?

— Stephen Hawking

The wavelength associated with an electron is related to the momentum of the electron by the de Broglie relation: $\lambda = \frac{h}{p}$

$$p = \frac{h}{\lambda} \longrightarrow \frac{dp}{dt} = \frac{p^2}{h} \times -\frac{d\lambda}{dt}$$

Sir Isaac Newton first presented his three laws of motion in the "**Principia Mathematica Philosophiae Naturalis**" in 1686. His second law defines a force exerted on the electron to be equal to the rate of change in momentum of the electron: $F = \frac{dp}{dt}$

$$F = \frac{p^2}{h} \times -\frac{d\lambda}{dt}$$

$$m_{relativistic} = \frac{m_e}{\sqrt{1-\frac{v^2}{c^2}}}$$

The mass of the electron is not constant; it varies with changes in its velocity.

$$m_{relativistic}^2 c^2 - m_{relativistic}^2 v^2 = m_e^2 c^2$$

On differentiation

$$m_{relativistic} v\, dv + v^2 dm_{relativistic} = c^2 dm_{relativistic}$$

$$dm_{relativistic}\,(c^2 - v^2) = m_{relativistic} v\, dv$$

$$\frac{dm_{relativistic}}{dt} = \frac{m_{relativistic} va}{(c^2 - v^2)}$$

$$m_{relativistic} c^2 = m_e c^2 + KE$$

$$\frac{dm_{relativistic} c^2}{dt} = \frac{dKE}{dt} = Fv$$

$$F = \frac{m_{relativistic} \times a}{1-\frac{v^2}{c^2}} = \frac{m_{relativistic}^3 a}{m_e^2}$$

For non-relativistic case (v << c):

$$F = m_e a$$

In no experiment, matter exists both as a particle and as a wave simultaneously. It is either the one or the other aspect.

$$F = \frac{m_{relativistic}^3 a}{m_e^2} = \frac{p^2}{h} \times -\frac{d\lambda}{dt}$$

$$a = \frac{m_e^2 v^2}{h m_{relativistic}} \times -\frac{d\lambda}{dt}$$

For nonrelativistic case (v << c):

$$a = \frac{m_e v^2}{h} \times -\frac{d\lambda}{dt}$$

Albert Einstein was a German-born theoretical physicist, widely acknowledged to be one of the greatest physicists of all time. Einstein is known for developing the theory of relativity, but he also made important contributions to the development of the theory of quantum mechanics.

"It was an act of desperation. For six years I had struggled with the blackbody theory. I knew the problem was fundamental and I knew the answer. I had to find a theoretical explanation at any cost, except for the inviolability of the two laws of thermodynamics."

− Max Planck

Irradiance is power per unit area.

Just like Energy, TOTAL MOMENTUM IS ALWAYS CONSERVED

Classical Picture	Quantum Picture
Energy of EM wave ~ (Amplitude) ^{2}	Energy of photon = $\frac{hc}{\lambda}$

The time will come when diligent research over long periods will bring to light things which now lie hidden. A single lifetime, even though entirely devoted to the sky, would not be enough for the investigation of so vast a subject... And so this knowledge will be unfolded only through long successive ages. There will come a time when our descendants will be amazed that we did not know things that are so plain to them... Many discoveries are reserved for ages still to come, when memory of us will have been effaced.

— Seneca

An 'up' quark has a charge of $+\frac{2}{3}$ e

$$q_{up} = +\frac{2}{3}\,e$$

and a 'down' quark has a charge of $-\frac{1}{3}$ e

$$q_{down} = -\frac{1}{3}\,e$$

$$F_E = \frac{q_{up}^2}{4\pi\varepsilon_0 r^2} = \frac{4\alpha\hbar c}{9r^2}$$

$$F_E = \frac{q_{down}^2}{4\pi\varepsilon_0 r^2} = \frac{\alpha\hbar c}{9r^2}$$

$$F_E = \frac{q_{up} \times q_{down}}{4\pi\varepsilon_0 r^2} = -\frac{2\alpha\hbar c}{9r^2}$$

$$\text{Hartree wave number} = \frac{1}{a_0} = \frac{\alpha^2}{r_e}$$

Hartree energy $= \hbar\omega_0 = 2\hbar c R_\infty = \alpha^2 m_e c^2$

$$\hbar\omega_0 = 2\hbar c R_\infty$$

Hartree frequency = 2 × Rydberg frequency

$$\hbar\omega_0 = \alpha^2 m_e c^2$$

Hartree frequency = α^2 **× Compton angular frequency of electron**

- Energy density of electric field = $\frac{\varepsilon_0 E^2}{2}$
- Energy density of magnetic field = $\frac{B^2}{2\mu_0}$

Electromagnetic wave consists of an oscillating electric field with a perpendicular oscillating magnetic field.

Energy density of EM wave:

"What is known of [photons] comes from observing the results of their being created or annihilated."

– Eugene Hecht

$$u_{wave} = \frac{\varepsilon_0 E^2}{2} + \frac{B^2}{2\mu_0}$$

$$c = \frac{E}{B} = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$$

$$u_{wave} = \varepsilon_0 E^2$$

u_{wave} does not depend on the frequency of the wave

$u_{particle}$ = number density of photons × hυ

depend on the frequency of the wave

$u_{wave} = u_{particle}$

number density of photons $\propto E^2$

"The more important fundamental laws and facts of physical science have all been discovered, and these are now so firmly established that the possibility of their ever being supplanted in consequence of new discoveries is exceedingly remote.... Our future discoveries must be looked for in the sixth place of decimals."

– Albert A. Michelson, 1894

Radiation pressure = $\frac{4\sigma T^4}{3c}$

A very small increase in temperature will result in a very large increase in the radiation pressure

Hydrostatic Equilibrium:

gas and radiation pressure balance the gravity

Thermal Equilibrium:

Energy generated = Energy radiated

$$\frac{\text{Hartree pressure}}{\text{Planck pressure}} = \frac{E_h}{a_0^3} \times \frac{L_{\text{Planck}}^3}{E_{\text{Planck}}}$$

Hartree pressure = $\alpha^5 \times (\text{electron gravitational coupling constant})^2 \times \text{Planck pressure}$

$$E_{\text{relativistic}}^2 = p^2c^2 + E_{\text{rest}}^2$$

$$E_{\text{relativistic}}^2 - E_{\text{rest}}^2 = p^2c^2$$

$$(E_{\text{relativistic}} - E_{\text{rest}})\,(E_{\text{relativistic}} + E_{\text{rest}}) = p^2c^2$$

$$KE = \frac{p^2}{(m_{\text{relativistic}} + m_{\text{rest}})}$$

For non-relativistic case:

$$KE = \frac{m_{\text{rest}}\, v^2}{2}$$

$$F = \frac{p^2}{h} \times -\frac{d\lambda}{dt} \longrightarrow F = \frac{KE(m_{\text{relativistic}} + m_{\text{rest}})}{h} \times -\frac{d\lambda}{dt}$$

$$KE = \frac{hF}{(m_{\text{relativistic}} + m_{\text{rest}}) \times -\frac{d\lambda}{dt}}$$

For non-relativistic case:

$$m_{\text{relativistic}} = m_{\text{rest}}$$

$$F = m_{\text{rest}}a$$

$$KE = \frac{ha}{2 \times -\frac{d\lambda}{dt}}$$

where K_J is the Josephson constant

$$KE = \frac{3k_BT}{2} = \frac{ha}{2 \times -\frac{d\lambda}{dt}}$$

$$a = \frac{3k_BT}{h} \times -\frac{d\lambda}{dt}$$

$$KE = eV = \frac{ha}{2 \times -\frac{d\lambda}{dt}}$$

$$a = K_JV \times -\frac{d\lambda}{dt}$$

Cherenkov radiation is the electromagnetic radiation emitted when a charged particle (such as an electron) travels in a medium with speed **v** such that:

$$\frac{c}{n} < v < c$$

where c is speed of light in vacuum, and n is the refractive index of the medium. We define the ratio between the speed of the particle and the speed of light as:

$$\frac{v}{c} = \frac{1}{n \times \cos\theta}$$

$$\cos\theta = \frac{c}{n \times v}$$

The emission of **Cherenkov radiation** depends on the refractive index n of the medium and the velocity v of the charged particle in that medium

The heavier the charged particle, the higher kinetic energy it must possess to be able to emit Cherenkov radiation.

Since the charged particle is relativistic, we can use the relation:

$$\lambda = \lambda_C \sqrt{\frac{c^2}{v^2} - 1}$$

$$\lambda = \lambda_C \sqrt{n^2\cos^2\theta - 1}$$

If $\lambda = \lambda_C$:

$$\theta = \cos^{-1}\left(\frac{\sqrt{2}}{n}\right)$$

The wavelength of the charged particle is equal to its Compton wavelength when Cherenkov angle equals $\cos^{-1}\left(\frac{\sqrt{2}}{n}\right)$

The **Cherenkov Effect** is used as a tool in:

- nuclear physics to detect solar neutrinos
- high energy experiments to identify the nature of particles
- astrophysical experiments to study the cosmic showers

Pavel Alekseyevich Cherenkov was a Soviet physicist who shared the Nobel Prize in physics in 1958 with **Ilya Frank** and **Igor Tamm** for the discovery of Cherenkov radiation, made in 1934.

"The element carbon can be found in more kinds of molecules than the sum of all other kinds of molecules combined. Given the abundance of carbon in the cosmos — forged in the cores of stars, churned up to their surfaces, and released copiously into the galaxy — a better element does not exist on which to base the chemistry and diversity of life. Just edging out carbon in abundance rank, oxygen is common, too, forged and released in the remains of exploded stars. Both oxygen and carbon are major ingredients of life as we know it."

— Neil deGrasse Tyson

For a spherical star of uniform density, the **gravitational binding energy** E_B is given by the equation:

$$E_B = -\frac{3GM^2}{5R}$$

where G is the gravitational constant, M is the mass of the star and R is its radius.

$$-\frac{E_B}{0.3Mc^2} = \frac{r_S}{R}$$

where $r_S = \frac{2GM}{c^2}$ is the Schwarzschild radius of the star. Any star with Radius smaller than its Schwarzschild radius will form a black hole.

If $R < r_S$:

$$|E_B| > 0.3Mc^2$$

The star will form a black hole

The **core pressure** of a star of mass M and radius R is given by:

$$P_{core} = \frac{5GM^2}{4\pi R^4}$$

↓

$$P_{core} = -\frac{25E_B}{9V} = -\frac{25}{9} \times \rho_B$$

where ρ_B is the gravitational binding energy density of the star.

Subrahmanyan Chandrasekhar was an Indian-American astrophysicist who spent his professional life in the United States. He was awarded the 1983 Nobel Prize for Physics with William A. Fowler for **"...theoretical studies of the physical processes of importance to the structure and evolution of the stars"**

$$\frac{10E_B}{3Mc^2} = -\frac{r_S}{R}$$

$-\frac{9P_{core}V}{25} = E_B$:

$$\frac{P_{core}}{0.833\rho_E} = \frac{r_S}{R}$$

where $\rho_E = \frac{Mc^2}{V}$ is the mass energy density of the star.

If $R < r_S$: $P_{core} > 0.833\rho_E$ — The star will form a black hole.

The **core density** of the star is given by:

$$\rho_{core} = \frac{3M}{\pi R^3}$$

The **core temperature** of the star is given by:

$$T_{core} = \frac{5\mu m_H GM}{3k_B R}$$

where k_B is the **Boltzmann constant**, μ denotes mean molecular weight of the matter inside the star and m_H is the mass of hydrogen nucleus

William Alfred Fowler was an American nuclear physicist, later astrophysicist, who, with Subrahmanyan **Chandrasekhar** won the 1983 Nobel Prize in Physics. He is known for his theoretical and experimental research into nuclear reactions within stars and the energy elements produced in the process.

$$\rho_{core} \times T_{core} = \frac{4\mu m_H P_{core}}{k_B} \longrightarrow P_{core} = \frac{\rho_{core} \times T_{core} \times k_B}{4\mu m_H}$$

$$\rho_B = -\frac{9P_{core}}{25} = -\frac{9 \times \rho_{core} \times T_{core} \times k_B}{100\mu m_H}$$

The ideal gas equation $PV = Nk_BT$ does not hold good for the matter present inside a star. Because, most stars are made up of more than one kind of particle and the gas inside the star is ionized. There is no indication of these facts in the above equation. We need to change the ideal gas equation, so that it holds good for the material present inside the star. It can be shown that the required equation can be written as $PV = \frac{M}{\mu m_H} k_BT$ where μ denotes mean molecular weight of the matter inside the star, M is the mass of the star and m_H is the mass of hydrogen nucleus.

$$\frac{PV}{MT} = \frac{k_B}{\mu m_H} = \frac{4P_{core}}{\rho_{core}T_{core}}$$

$$\frac{P}{P_{core}} = 4 \times \frac{\rho}{\rho_{core}} \times \frac{T}{T_{core}}$$

$$\textbf{Planck force density} = \frac{\text{Planck force}}{\text{Planck volume}} = \frac{\text{Planck pressure}}{\text{Planck length}} = \sqrt{\alpha}\ \frac{\text{Planck pressure}}{\text{Stoney length}}$$

$$m_e c^2 = \frac{G m_p m_e}{r}$$

$$r = \frac{\text{Schwarzschild radius of proton}}{2}$$

Distance between proton and electron at which the gravitational potential energy between them is equal to intrinsic energy of electron

The saddest aspect of life right now is that science gathers knowledge faster than society gathers wisdom.

— **Isaac Asimov**

$$m_p c^2 = \frac{G m_p m_e}{r}$$

$$r = \frac{\text{Schwarzschild radius of electron}}{2}$$

Distance between proton and electron at which the gravitational potential energy between them is equal to intrinsic energy of proton

Black hole type	Description	Constraints	
Schwarzschild	has no angular momentum and no electric charge	angular momentum = 0	electric charge = 0
Kerr	does have angular momentum but no electric charge		electric charge = 0
Reissner–Nordström	has no angular momentum but does have an electric charge	angular momentum = 0	
Kerr–Newman	has both angular momentum and an electric charge		

$$\frac{Mc^2}{2} = T_{BH} \times S_{BH}$$

Specific heat capacity of a black hole $= -\frac{2S_{BH}}{M}$

Heat Capacity: $C = \frac{dQ}{dT}$

Substituting $dQ = dMc^2$ and $T = \frac{\hbar c^3}{8\pi k_B GM}$:

Heat capacity of a black hole $= -\frac{8\pi k_B GM^2}{\hbar c}$

Specific heat capacity of a black hole $= -\frac{8\pi k_B GM}{\hbar c} = -\frac{c^2}{\text{Black hole temperature}}$

$$S_{BH} = \frac{4\pi k_B GM^2}{\hbar c}$$

On differentiation

$$dS_{BH} = \frac{8\pi k_B GM}{\hbar c^3} \times dMc^2$$

$$\left\{ T_{BH} \times dS_{BH} = dMc^2 \right\}$$

"For the past forty-five years, Stephen and hundreds of other physicists have struggled to understand the precise nature of a black hole's randomness. It is a question that keeps on generating new insights about the marriage of quantum theory with general relativity—that is, about the ill-understood laws of quantum gravity."

— Stephen Hawking

Black holes are the harmonic oscillator of quantum gravity.

(A. Strominger)

$$\frac{Mc^2}{2} = T_{BH} \times S_{BH}$$

On differentiation

$$dMc^2 = 2\,(T_{BH} \times dS_{BH}) + 2\,(dT_{BH} \times S_{BH}) = 2\,dMc^2 + 2\,(dT_{BH} \times S_{BH})$$

$$-\frac{dMc^2}{dT_{BH}} = 2S_{BH}$$

- Neutron Star has a hard surface; the curvature is large - but finite.
- **Black Hole:** No Surface − curvature is infinite at the centre.

A photon of higher frequency causes the ejected photoelectron to propagate faster. The energy of photon − converted into the kinetic energy of the electron − is proportional to its frequency.

It is impossible, using the current laws of quantum mechanics and the known behavior of gravity, to determine a position to a precision smaller than $\sqrt{\frac{\hbar G}{c^3}}$

$\hbar$, c, G, e, ε_0, m_e, m_p …..

Other constants

$$\lambda_C = \frac{h}{m_p c}$$

$$r_e = \frac{e^2}{4\pi\varepsilon_0 m_e c^2}$$

Fundamental dimensionless constants

$$\frac{m_p}{m_e}$$

$$\alpha = \frac{e^2}{4\pi\varepsilon_0\, \hbar c}$$

$$\alpha_G = \frac{G m_e^2}{\hbar c}$$

Magnetic coupling constant = $\frac{1}{4\alpha}$

$\hbar$, c, G, ε_0

Planck units

$$m_{Planck} = \sqrt{\frac{\hbar c}{G}}$$

$$L_{Planck} = \sqrt{\frac{\hbar G}{c^3}}$$

$$t_{Planck} = \sqrt{\frac{\hbar G}{c^5}}$$

$$q_{Planck} = \sqrt{4\pi\varepsilon_0 \hbar c}$$

$$\frac{m_p}{m_e} = \frac{\mu_B}{\mu_N} = \sqrt{\frac{\text{proton gravitational coupling constant}}{\text{electron gravitational coupling constant}}}$$

The **Planck units** simplify the expression of physics laws and are the universal limits beyond which all the known laws of physics break down. In order to comprehend anything beyond it − we need new unbreakable laws of theoretical physics.

Theories of proton decay predict that the proton has a half life on the order of at least 10^{32} years. Till date, there is no experimental evidence of proton decay.

If you wish to make an apple pie from scratch, you must first invent the universe.

— **Carl Sagan**

$$\mu_B \times r_S = \frac{e\hbar}{2m_e} \times \frac{2Gm_e}{c^2}$$

$$\left\{ \mu_B \times r_S = \frac{\sqrt{\alpha} \times c_1 \times q_{Planck}}{4\pi^2 \times F_{Planck}} \right\}$$

$$\mu_N \times r_S = \frac{e\hbar}{2m_p} \times \frac{2Gm_p}{c^2} = \frac{\sqrt{\alpha} \times c_1 \times q_{Planck}}{4\pi^2 \times F_{Planck}}$$

A thinker sees his own actions as experiments and questions--as attempts to find out something. Success and failure are for him answers above all.

— Friedrich Nietzsche

$$\mu_B \times a_0 = \frac{e\hbar}{2m_e} \times \frac{\hbar}{m_e c\alpha}$$

$$\mu_B \times a_0 = \frac{eQ_0^2}{2\pi^2 c\alpha}$$

$$\left\{ \mu_B \times r_e = \frac{\alpha e Q_0^2}{2\pi^2 c} \right\}$$

$$F_G = \frac{Gm_e^2}{r^2} = \frac{GE_{rest}^2}{c^4 r^2} = \frac{k_B^2 T_{threshold}^2}{\text{Planck force} \times r^2}$$

$$F_G = \frac{1}{4\pi^2 \times \text{Planck force}} \times \frac{(\text{First radiation constant})^2}{(\text{Second radiation constant})^2} \times \frac{\mu_0 \varepsilon_0 T_{threshold}^2}{r^2}$$

$$F_G = \frac{Gm_{Planck}^2}{r^2} = \frac{1}{4\pi^2 \times \text{Planck force}} \times \frac{(\text{First radiation constant})^2}{(\text{Second radiation constant})^2} \times \frac{\mu_0 \varepsilon_0 T_{Planck}^2}{r^2}$$

$$F_G = \frac{Gm_e^2}{r^2} = \frac{GE_{rest}^2}{c^4 r^2} = \frac{h^2\upsilon_C^2}{\text{Planck force} \times r^2}$$

$$F_G = \frac{c_1 \times \text{Planck angular momentum}}{\text{Planck force} \times \lambda_C \times r^2}$$

$$F_G = \frac{Gm_e^2}{r^2} = \frac{G}{r^2} \times \frac{e^2\hbar^2}{4\mu_B^2}$$

$$F_G = \frac{L_{Planck}^2 e^2 c_1}{16\pi^2\mu_B^2\, r^2\sqrt{\varepsilon_0\mu_0}} = \frac{\text{Planck area} \times e^2 \times c_1}{16\pi^2 \times \mu_B^2 \times r^2 \times \sqrt{\varepsilon_0\mu_0}}$$

Gravitational redshift

The change in the wavelength of electromagnetic radiation photon in a gravitational field predicted by general theory of relativity. A heuristic Newtonian derivation gives

$$z = \frac{\Delta E}{E} = -\frac{GM}{rc^2}$$

$$Mc^2 = -z \times \text{Planck force} \times r$$

I do not feel obliged to believe that the same God who has endowed us with sense, reason, and intellect has intended us to forgo their use.

— Galileo Galilei

[Letter to the Grand Duchess Christina]

Science, my lad, is made up of mistakes, but they are mistakes which it is useful to make, because they lead little by little to the truth.

— Jules Verne

Gravitational waves are 'ripples' in space-time, generated by accelerated masses that propagate as waves outward from their source at the speed of light. They were proposed by **Henri Poincaré** (French mathematician, theoretical physicist, engineer and philosopher of science) in 1905 and subsequently predicted in 1916 by **Albert Einstein** on the basis of his general theory of relativity.

Gravitational waves were first directly detected by the Laser Interferometer Gravitational-Wave Observatory (LIGO) in 2015. **Gravitational wave** is to gravity what light is to electromagnetism. It is the transmission of variations in the gravitational field as waves. Predicted by Einstein's theory of general relativity, the waves transport energy known as gravitational radiation. Two objects orbiting each other in highly elliptical orbit or circular orbit about their center of mass comprises binary system. This system loses mass by emitting gravitational wave (**ripple in the geometry of space and time**) whose frequency $\upsilon = \frac{E}{h} <<$ frequency of electromagnetic radiation and this is associated with an in-spiral or decrease in orbit. Suppose that the two masses are m_1 and m_2, and they are separated by a distance "r" orbiting each other in highly circular orbit about their center of mass. The rate of loss of energy from the binary system through gravitational radiation is given by:

$$P = -\frac{dE}{dt} = \frac{32G^4\, m_1^2 m_2^2}{5c^5}\,\frac{(m_1+m_2)}{r^5}$$

where $G = 6.674 \times 10^{-11}\ m^3\ kg^{-1}\ s^{-2}$ is the **Newtonian gravitational constant** and $c = 3 \times 10^8\ ms^{-1}$ is the **speed of light in vacuum**. Gravitational radiation robs the energy of orbiting masses. As the energy of the orbiting masses reduces, the distance between the masses decreases, and they orbit more rapidly. More generally, the rate of decrease of distance between the masses with time is given by:

$$v = -\frac{dr}{dt} = \frac{64G^3\, m_1 m_2}{5c^5}\,\frac{(m_1+m_2)}{r^3}$$

$$P = v \times \frac{Gm_1m_2}{2r^2}$$

$$\left\{ F_G = \frac{2P}{v} \right\}$$

where F_G is the force of gravitation between the two masses orbiting each other in highly circular orbit about their center of mass. The loss of energy through **gravitational radiation** could eventually drop the mass m_1 into the mass m_2. The **lifetime of distance** "r" between the masses orbiting each other in highly circular orbit about their center of mass is given by:

$$t_{life} = \frac{5c^5r^4}{256G^3m_1m_2\,(m_1+m_2)}$$

$$t_{life} = \frac{r}{4 \times v}$$

$$F_G = \frac{2P}{v} = \frac{8P \times t_{life}}{r}$$

The **gravitational wave signal** was observed by LIGO detectors in Hanford and in Livingston on 14 **September 2015**. An exact analysis of the gravitational wave signal based on the **Albert Einsteinian theory of general relativity** showed that it came from two merging stellar black holes with 29 and 36 solar masses, which merged 1.3 billion light years from Earth. Before the merger, the total mass of both black holes was 36 + 29 solar masses = 65 solar masses. After the merger, the mass of resultant black hole was 62 solar masses.

What happened to three solar masses?

It was turned into the energy transported by the emitted gravitational waves. Using Albert Einstein's equation $E = mc^2$, where E is the energy transported by the emitted gravitational waves, m is the missing mass (3 solar masses) and c is the speed of light, we can estimate the energy released as **gravitational waves**:

The amplitude of gravitational waves gets smaller with the distance to the source.

$$E = (3 \times 2 \times 10^{30}\ \text{kg}) \times (3 \times 10^{8}\ \text{m/s})^{2}$$

$$E = 5.4 \times 10^{47}\ \text{J}$$

This is roughly 10^{21} more energy than the complete electromagnetic radiation emitted by our sun.

$$\upsilon = \frac{E}{h} = \frac{5.4 \times 10^{47}}{6.626 \times 10^{-34}} = 8.14 \times 10^{80} s^{-1}$$

- **Gravity** → Curvature of 4-dimensional (3 space + 1 time) space-time fabric produced by matter.
- **Gravitational-waves** → Ripples on 4-dimensional space-time produced by accelerated matter.

"**Newton's law of gravitation**. That's all you need (with a spot of calculus to crunch the numbers) to work out how the Earth will orbit the Sun or how an apple will fall if you let it go at a certain height. The only trouble is that Newton had no idea how this gravity thing worked. His model was simply: There is an attraction between bits of stuff, and let's not bother about why."

— Brian Clegg

Albert Einstein theorized that smaller masses travel toward larger masses, not because they are "attracted" by a mysterious force called gravity, but because the smaller objects travel through space that is warped by the larger object.

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- Hawking on the Big Bang and Black Holes by **Stephen W. Hawking.**
- The Universe in a Nutshell by **Stephen W. Hawking**.

Natural science, does not simply describe and explain nature; it is part of the interplay between nature and ourselves.

Werner Heisenberg