

IS THE UNIVERSE MADE OF MATH?

# Mathematical Universe

MANJUNATH.R

# Mathematical Universe:

**Our Search for the Ultimate Nature of Reality**



"The only true wisdom is in knowing you know nothing."

– Socrates

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For any suggestions or concerns, please write to me: [manjunath5496@gmail.com](mailto:manjunath5496@gmail.com)

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## Dedication

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I **dedicate** this book to everyone who has contributed significantly to our understanding of the universe as a whole, why it is the way it is, and why it even exists.

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## Acknowledgements

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Without the amazing work of some renowned cosmologists and physicists, their creativity, and their inventiveness in the field of cosmology, this book would not have been accomplished. I would like to use this opportunity to thank my dearest friend and well-wisher "**Lawrence**" for his unwavering support during the **COVID crisis** and for giving me access to all the resources I needed to finish this book. I want to express my gratitude to my family for their support and encouragement as I wrote this book, especially to my **mother**, who has been a tremendous source of inspiration in my life. I owe a lot of gratitude to my mother for teaching me how to be perseverant and strong in life. Finally, I want to emphasize how crucial patience is when writing a book or taking on any other project in life.

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"My goal is simple. It is a complete understanding of the universe, why it is as it is and why it exists at all."

– Stephen Hawking



## Introduction:

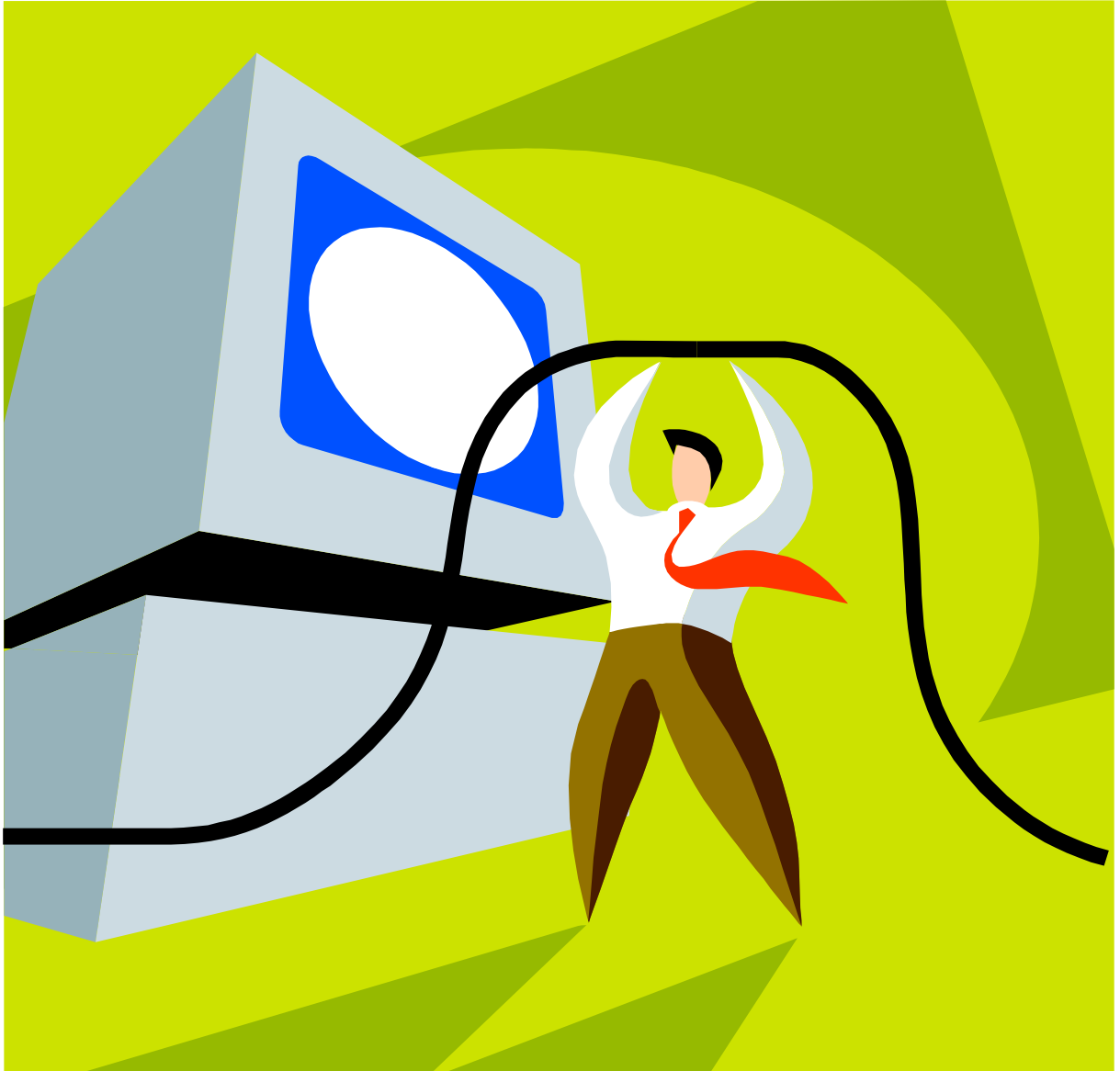
With the help of exact astronomical observations and **geometry** (a field of mathematics that examines objects' shapes, positions, angles, and lengths), the paths of the heavenly bodies were mathematically precisely described, and they remarkably corresponded to what we can observe. It is true that mathematics gives us the ability to statistically describe the Universe; this makes it a very helpful tool. The idea that our cosmos is mathematical in some way dates at least as far back as the **Greek Mathematicians** and has received a great deal of attention in the scientific literature that offers solutions to our universe-related questions. What is our origin? What gives the universe its current form? Basically, why are we here? **Albert Einstein's 1915 general relativity theory**, which states that the gravitational force results from the curvature of space and time, is our best explanation of how gravity operates. In order to explore the mysteries disclosed by cosmos and determine the nature of reality, **Einstein** had to develop this theory using mathematics. We can better understand our world and everything around us by using mathematics, which reveals hidden patterns. Our world is built on mathematics, which may be observed in amazing ways. The shape of a regular hexagon, which has six equal sides, is frequently encountered in the nature. A **bee hive** is the most prevalent example of a hexagonal structure in nature. A group of concentric circles is another typical shape found in nature. When something strikes the **water's surface**, it causes ripples that seem like concentric circles.

**Eugene Wigner** – a Hungarian-American theoretical physicist whose "contributions to the theory of the atomic nucleus and the elementary particles, particularly through the discovery and application of fundamental symmetry principles" earned him the Nobel Prize in Physics in 1963 – conveyed on the "unreasonable effectiveness of mathematics in the natural sciences," and Italian astronomer **Galileo Galilei** insisted that the universe is a big book written in the language of mathematics. We shall take this idea to its logical conclusion in this book and contend that our cosmos is mathematical structure in a clear-cut sense. **Mathematics not only explains but also defines physical reality.** Albert Einstein spent his whole life looking for a single, groundbreaking "math equation" that would unite all the known forces in the cosmos, but he was never

successful. From **Stephen William Hawking** to **Brian Randolph Greene**, some of the brightest minds in physics took up the pursuit. No one has done so as of now.

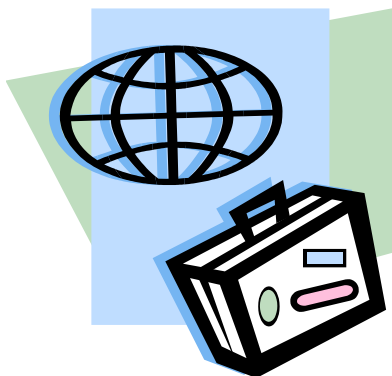






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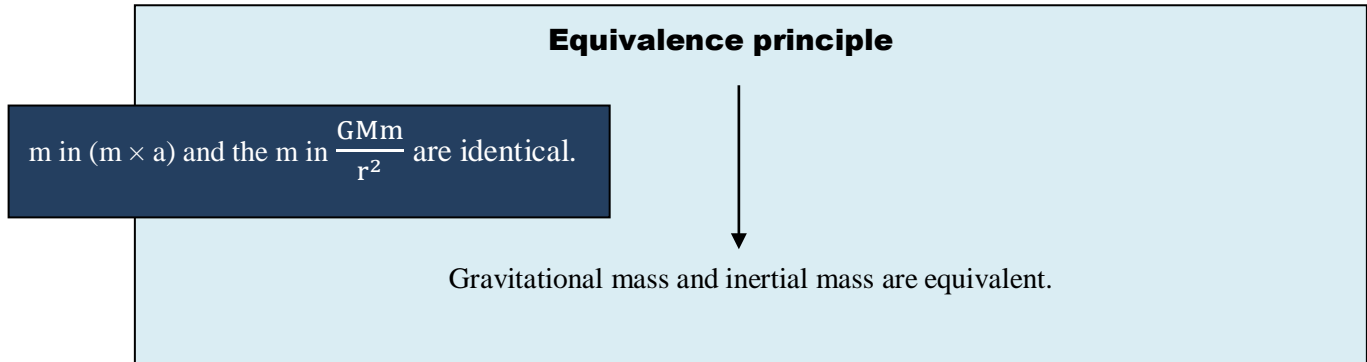
**Science knows no country, because knowledge belongs to humanity, and is the torch which illuminates the world. Science is the highest personification of the nation because that nation will remain the first which carries the furthest the works of thought and intelligence.**

Louis Pasteur

**"Our universe may be one of an infinite number of parallel universes, each connected to the others by an infinite series of wormholes."**

### **3 Laws of Universe:**

- You cannot get something for nothing because matter and energy are conserved.
- You cannot return to the same energy state because there is always an increase in entropy.
- Absolute zero is unattainable.



Matter-energy  $\rightarrow$  curvature of spacetime

(Spacetime is curved by the presence of matter-energy)

### **Gravity is described by curvature of spacetime**

Flat spacetime  $\rightarrow$  Zero curvature of spacetime

Zero curvature of spacetime  $\rightarrow$  Zero gravity

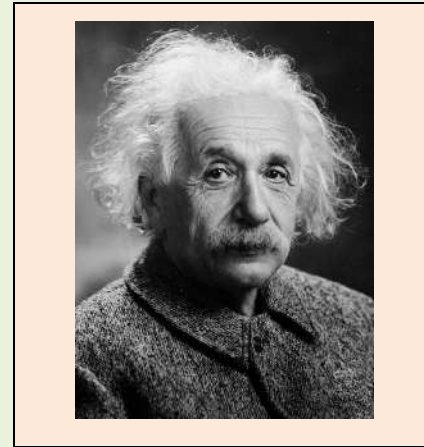
Geometry  $\rightarrow$  Field Theory  $\rightarrow$  Classical Theory  $\rightarrow$  Quantum Theory

"But the creative principle resides in mathematics. In a certain sense, therefore, I hold it true that pure thought can grasp reality, as the ancients dreamed."

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

↓

[Mass changes as a function of velocity]



– Albert Einstein

**Newton's laws of motion tie into almost everything we see in everyday life.**

- **Law 1 (Law of inertia):** An object at rest stays at rest and an object in motion stays in motion with the same speed and in the same direction unless acted upon by an unbalanced force.
- **Law 2:** Force equals mass times acceleration ( $F = ma$ ).
- **Law 3:** For every action, there is an equal and opposite reaction.

As a remarkable consequence of the Heisenberg's uncertainty principle of quantum mechanics (which implies that certain pairs of quantities, such as the energy and time, cannot both be predicted with complete accuracy), the empty space is filled with what is called vacuum energy.

**Because  $E=mc^2$ :**

- Mass is just energy in disguise.

**In relativity, there is no such thing as an absolute time or absolute length.**

- A small amount of mass can equal a large amount of energy.

$$E = mc^2$$



The removal of energy is the same as the removal of mass

#### If the mass of the star < 1.4 solar masses

- Electrons prevent further collapse.
- The core will thus continue to collapse and form a white dwarf.

#### If the mass of the star > 1.4 solar masses but mass < 3 solar masses

- Electrons + protons combine to form neutrons.
- Neutrons prevent further collapse.
- The core will thus continue to collapse and form a neutron star.

#### If the mass of the star > 3 solar masses

- Gravity wins! Nothing prevents collapse.
- The core will thus continue to collapse and form a black hole.

As we know,  $c^2$  is huge:  $9 \times 10^{16}$  meters square per Second Square. So a small amount of uranium or plutonium can produce such a massive atomic explosion.

**Any object with a physical radius smaller than its Schwarzschild radius will be a black hole.**

### Did you know?

A body in motion moves in a straight path at a constant speed until acted upon by an external force, while a body at rest remains at rest in an inertial frame of reference.



## The Principle of Relativity



All the **laws of physics** are the same in all inertial frames of reference. For example: If we use the characteristics of one inertial frame of reference to define a physical law, the resulting statement of the law will be the same in every other inertial frame of reference.

## The Principle of Invariant Light Speed



**Light** always moves with a fixed velocity "c" ( $3.0 \times 10^8 \text{m/s}$ ) in empty space, independent of the state of motion of the emitting body.

$$[\text{Force}] = \frac{[\text{change in momentum}]}{[\text{time duration}]}$$

If [time duration]  $\rightarrow$  decreases,

then [Force]  $\rightarrow$  increases

$$[\text{impulse}] = [\text{Force}] \times [\text{time duration}]$$



$$[\text{impulse}] = [\text{change in momentum}]$$

The greater the impulse, the greater the change in momentum.



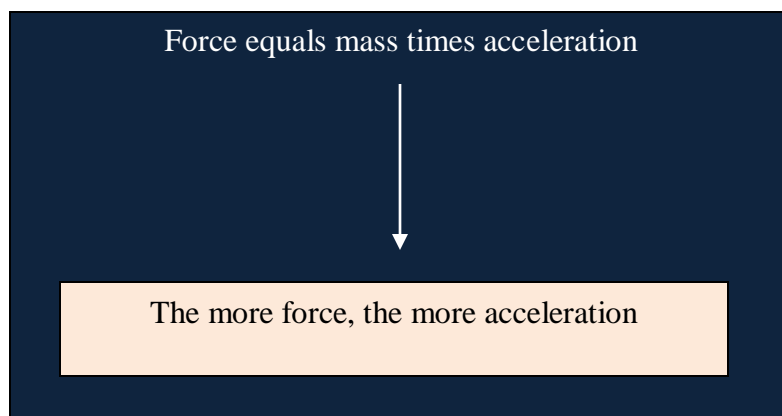
**Sir Isaac Newton**

## Newton's Second Law of Motion:

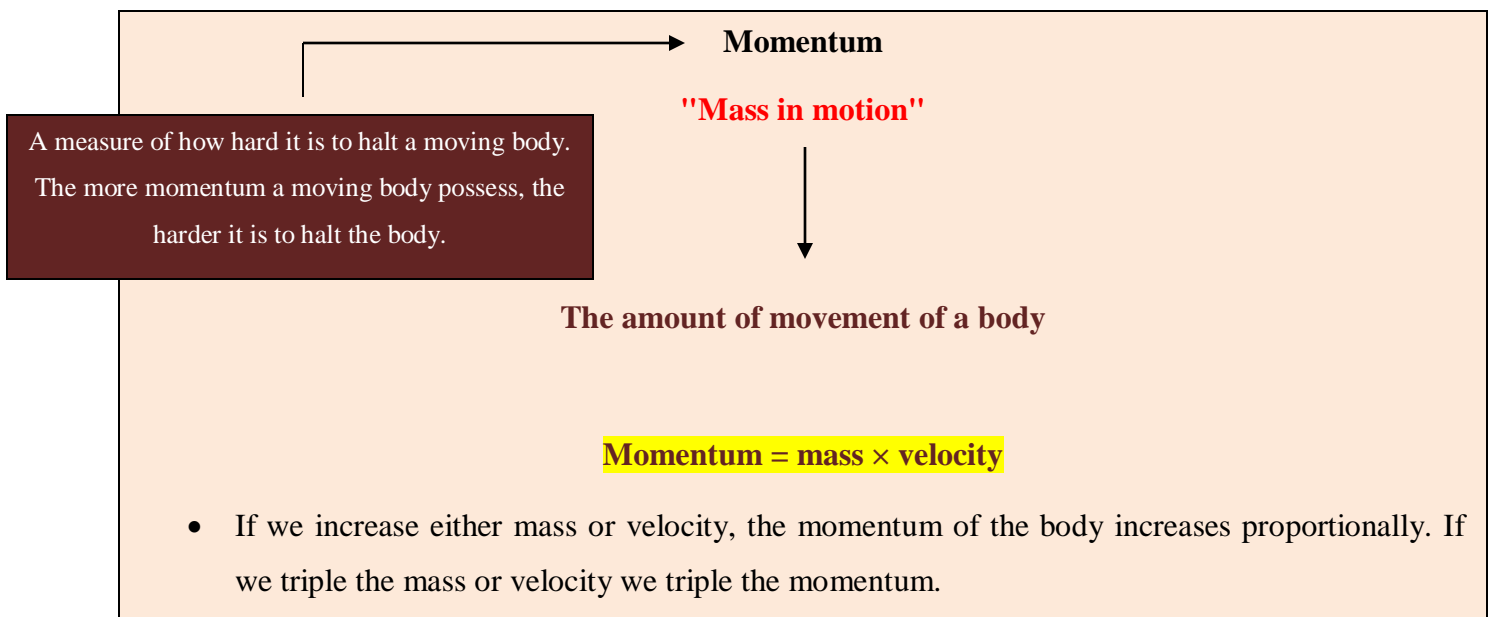
Force is equal to the rate of change of momentum.

i.e., Momentum gained = Force  $\times$  Duration through which force acts

For a constant mass:



**\*In relativistic mechanics:** Force = mass  $\times$  acceleration no longer remain valid.





## Inertia



The tendency of a body to resist changes in its state of motion

A body's inertia increases linearly with its mass, hence it requires more force to change its state of motion. A massive body is more likely to withstand changes in its state of motion.

|  |   |
|--|---|
| <b>Mass</b><br>↓<br>A measure of the amount of matter in an object | <b>Weight</b><br>↓<br>A measure of the gravitational force acting on the object |
|--|---|

$$\text{Weight} = \text{mass} \times \text{gravitational acceleration}$$

The weight of a body changes with gravitational acceleration while the mass of a body remains constant. As a result, weight varies although mass remains constant in different locations.

|                      |   |
|----------------------|---|
| <b>Mass on Earth</b> | <b>Weight on Earth</b>                                  |
| 1 kg                 | $1\text{kg} \times 9.81 \text{ m/s}^2 = 9.81 \text{ N}$ |
| <b>Mass on Moon</b>  | <b>Weight on Moon</b>                                   |
| 1 kg                 | $1\text{kg} \times 1.62 \text{ m/s}^2 = 1.62 \text{ N}$ |

Massless particles have no rest mass energy – just kinetic energy



The energy resulting from a particle's motion

|  |  |
|--|--|
| <p><b>Speed of light</b></p> <p>(constant of special relativity)</p>           | $c = 3.0 \times 10^8 \text{ m/s}$  |
| <p><b>Gravitational constant</b></p> <p>(constant of gravity)</p>              | $G = 6.673 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$               |
| <p><b>Reduced Planck's constant</b></p> <p>(constant of quantum mechanics)</p> | $\hbar = \frac{h}{2\pi} = 1.054571596 \times 10^{-34} \text{ kg m}^2 \text{ s}^{-1}$ |
| <p><b>Boltzmann constant</b></p> <p>(constant of thermodynamics)</p>           | $k_B = 1.3806503 \times 10^{-23} \text{ kg m}^2 \text{ s}^{-2} \text{ K}^{-1}$       |

$$\text{Planck Length} = \sqrt{\frac{\hbar G}{c^3}} = 1.62 \times 10^{-35} \text{ m}$$

$$\text{Planck Time} = \sqrt{\frac{\hbar G}{c^5}} = 5.39 \times 10^{-44} \text{ s}$$

$$\text{Planck Temperature} = \sqrt{\frac{\hbar c^5}{G k_B^2}} = 1.41 \times 10^{32} \text{ K}$$

$$\text{Planck energy} = \sqrt{\frac{\hbar c^5}{G}} = 1.9561 \times 10^9 \text{ J}$$

At this energy, all the known four fundamental forces of nature — gravity, the weak force, electromagnetism and the strong force — finally merge into one force.

**All the laws of physics that we know, breaks down –**

- Below this time: (Planck Time)
- Below this length: (Planck Length)
- Above this temperature: (Planck Temperature)

New, unbreakable physics is required.

## Did you know?

At the Planck scale, the gravitational fluctuations resulting from quantum fluctuations could cause space-time to become quantized and chaotic quantum foam.

$$\text{Planck Mass} = \sqrt{\frac{\hbar c}{G}} = 2.18 \times 10^{-8} \text{ kg}$$

### Why the Planck mass is so large?



Because the gravitational force is very weak

$$(G = 6.673 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2})$$

**If a black hole has a mass less than the **Planck mass**, its quantum mechanical size could be outside its event horizon. This wouldn't make sense, **Planck mass** is the smallest possible black hole.**

$$\text{Refractive Index} = \frac{\text{speed of light in vacuum}}{\text{speed of light in matter}}$$

$$n = \frac{c}{v}$$

Since  $c$  is constant,  $n$  is inversely proportional to  $v$ :

$$n \propto \frac{1}{v}$$

The higher the refractive index, the slower the light travels in matter.

## Quantum chromodynamics binding energy



### The energy binding quarks together into hadrons

Because:

$$\text{Schwarzschild radius} = \frac{2 \times \text{Gravitational constant} \times \text{mass of the object}}{(\text{Speed of light})^2}$$

Any object with mass equal to that of our Sun to be a black hole, its radius would be about 3 km

- **Escape velocity** is the speed that is required to leave the orbit of a planet or star

$$v_{\text{escape}} = \sqrt{\frac{2GM}{R}}$$

- **Orbital velocity** is the speed that is required to remain in the orbit of a planet or star.

$$v_{\text{orbital}} = \sqrt{\frac{GM}{R}}$$

$$\text{Escape velocity} = \sqrt{2} \times \text{Orbital velocity}$$



The escape velocity will increase if the orbital velocity increases, and vice versa.

Due to the enormous gravitational field within the event horizon, it is believed to be impossible to escape from a black hole because the escape velocity exceeds the speed of light.

$$F_G = \frac{GMm}{r^2}$$

Because G is small, gravitational force is very small unless large masses are involved.

where  $F_G$  is the attractive gravitational force between the masses M and m which are separated by a distance r and G is the constant of gravitation.

As "r" becomes smaller, " $F_G$ " becomes larger, which means that the masses M and m move toward one another with increasing acceleration.

**Density parameter ( $\Omega$ ):** The ratio of the total amount of matter in the universe divided by the minimum amount of matter needed to cause the big crunch.

- **$\Omega < 1$ :** The Universe will continue to expand forever.
- **$\Omega > 1$ :** The Universe will eventually halt its expansion and recollapse.
- **$\Omega = 1$ :** The Universe contains enough matter to halt the expansion but not enough to recollapse it.

### Absolute zero

**(-273°C)**



The lowest possible temperature, at which substances contain no heat energy and all vibrations stop — almost.

If the two quarks would have occupied precisely the same point with the same properties, they would not have stayed in the same position for long. And quarks would have not formed separate, **well-defined protons and neutrons**. And nor would these, together with electrons have formed separate, well-defined atoms. And the world would have collapsed before it ever reached its present size.

When 2 similar waves are added, the resultant wave is bigger



**(constructive interference)**

When 2 dissimilar waves are added, they cancel each other out



**(destructive interference)**

**The different frequencies of light appear as different colors.**

| Frequency (Terahertz) | Color  |
|-----------------------|--------|
| 400 - 480             | Red    |
| 480 - 510             | Orange |
| 510 - 530             | Yellow |
| 530 - 600             | Green  |
| 600 - 670             | Blue   |
| 670 - 700             | Indigo |
| 700 - 750             | Violet |

**Proton charge + Electron charge = 0**



Just what it is if **electromagnetism** would not dominate over **gravity** and for the universe to remain electrically neutral.

- It's not their energy; it's their zero rest mass that makes **photons** to travel at the speed of light.
- Just like a dozen is 12 things, a mole is simply **Avogadro's number of particles**.

Undisturbed space + rigid mass → distorted space

$$h \rightarrow 6.62607015 \times 10^{-34} \text{ J s}$$



Since  $h$  is incredibly small, the frequency of the photon is always greater than its energy, so it do not take many quanta to radiate even ten thousand megawatts.

### What is GRAVITY?



- **Newtonian view:** Force tells mass how to accelerate. Accelerated mass tells what gravity is.
- **Einsteinian view:** Mass tells space how to curve. Curved space tells what gravity is.

All objects emit electromagnetic radiation according to their temperature. Colder objects emit waves with very low frequency (such as radio or microwaves), while hot objects emit visible light or even ultraviolet and higher frequencies.

- Longer half-life of nucleus → **Slow Radioactive Decay.**
- **Shorter half-life of nucleus** → Fast Radioactive Decay.

### MATTER UNDER EXTREME CONDITIONS



Nuclei + heat + pressure → quark-gluon plasma

## ".. Physics at the atomic and subatomic level ..."

... Weird things are possible:

- Energy is quantized ( $E = nh\nu$ )
- Momentum is quantized ( $L = n\hbar$ )
- Charge is quantized ( $Q = ne$ )

When we observe how objects move around in gravitationally curved trajectories in space, we discover another recurring shape: **the ellipse.**

### Emission of electron:

- Conversion of a neutron into a proton
- Atomic number increases by 1, Number of neutrons decreases by 1
- Atomic mass number unchanged

### Emission of positron:

- Conversion of a proton into a neutron
- Number of neutrons increases by 1, Atomic number decreases by 1
- Atomic mass number unchanged

### Emission of a neutron:

- Number of neutrons decreases by 1
- Atomic number unchanged
- Atomic mass number decreases by 1

**Because:**

$$E = h\nu, c = \lambda\nu, E = \frac{hc}{\lambda} = pc$$



$$\lambda = \frac{h}{p}$$

$$\lambda \propto \frac{1}{p}$$



With increasing momentum, the wavelength approaches zero, while with decreasing momentum, the wavelength increases without bound

**Every particle or quantum entity may be partly described in terms not only of particles, but also of waves.**

### **The Thermodynamic Laws think big: they dictate energy behavior...**

- **1 Law:** Energy is conserved; its form can be converted.
- **2 Law:** Energies can flow, equilibrate.
- **3 Law:** "Driving force" for equilibration uniquely defined.
- **0 Law:** Thermal equilibrium is transitive.

### **String Theory**



A theory that tries to adjust or harmonize or reconcile **General Relativity** and **Quantum mechanics**

- Different vibrations → Different particles.
- String combinations → Particle interactions.

## The Life of a Star:

"More mass  
More pressure and temperature  
Faster Fusion  
Shorter life"

"Less mass  
Less pressure and temperature  
Slower Fusion  
Longer life"

- **Hydrogen atom:** Diameter about a Billionth of an inch.
- **Electron:** Diameter at least 1000 times smaller than that of proton.
- **Proton:** Diameter about 60,000 times smaller than Hydrogen atom.
- **Probability distribution** is the only way to locate an electron in an atom.

## The Gas laws deal with how gases behave with respect to pressure, volume, temperature ...

- **Boyle's law:**

Volume and pressure are inversely proportional.

(If the temperature of a gas is held constant, increasing the pressure of the gas decreases its volume)

- **Charles' law:**

Volume is proportional to temperature.

(If the pressure of a gas is held constant, increasing the temperature of the gas increases its volume)

- **Pressure law:**

Pressure is proportional to temperature.

(If the volume of a gas is held constant, increasing the temperature of the gas increases its pressure)

All three combined:

$$\frac{\text{Pressure} \times \text{Volume}}{\text{Temperature}} = \text{constant}$$

- **Stable nucleus** → non-radioactive
- **Unstable nucleus** → radioactive
- **Less stable nucleus** → more radioactive
- **More stable nucleus** → less radioactive

**Because:**

$$\text{Time period} = \frac{1}{\text{Frequency}}$$

**The time period of a wave increases as its frequency decreases.**

$$\text{Binding Energy per nucleon} = \frac{\text{Binding Energy}}{\text{Nucleon Number}}$$

**Since it requires more energy to pull the nucleus apart:**

**A higher binding energy per nucleon → The more stable the nucleus**

**Nucleus** + **Binding energy** → **Separated nucleons**  
(Smaller mass) (Greater mass)

Mass of a stable nucleus is always less than the sum of masses of its constituent protons and neutrons in their free state.

The energy needed to disassemble the nucleus into its constituent protons and neutrons.

$$\text{Binding energy} = [\text{mass difference}] \times c^2$$

**Greater the mass difference, greater will be the binding energy**

(The most stable nuclei are those with the highest binding energies)

Weak nuclear forces + **Maxwell equations** → Electro weak theory

Electro weak theory + Quantum Chromodynamics (**QCD**) → Standard Model of particle physics

**Standard Model** of particle physics → explains everything except **gravity**

#### **4 NUMBERS that describe the characteristics of electrons and their orbitals:**

- **Principal quantum number:** A number that describes the average distance of the orbital from the nucleus and the energy of the electron in an atom.
- **Angular momentum quantum number:** A number that describes the shape of the orbital.
- **Magnetic quantum number:** A number that describes how the various orbitals are oriented in space.

- **Spin quantum number:** A number that describes the direction the electron is spinning in a magnetic field — either clockwise or counterclockwise.

### **Kepler's Third Law**

The square of the periods of the planets is proportional to the cubes of their average distance from the Sun.



A consequence of this is that the **inner planets** move rapidly in their orbits. Venus, Earth and Mars move progressively less rapidly about the Sun. And the **outer planets**, such as Jupiter and Saturn, move stately and slow.

Wavelength of UV radiation < Wavelength of IR radiation < Wavelength of microwave radiation

- Molecule dissociates (**when it absorbs UV radiation**).
- Molecule vibrates (**when it absorbs IR radiation**).
- Molecule rotates (**when it absorbs microwave radiation**).

**If the expansion of space had overwhelmed the pull of gravity in the beginning – stars, galaxies and humans would never have been able to form. If, on the other hand, gravity had been 5% stronger– stars and galaxies might have formed, but they would have quickly collapsed in on themselves and each other to form a sphere of roughly infinite density.**

## Neutrons have a mass of 939.56 MeV



If the mass of a neutron was a seventh of a percent more than it is, stars like most of those we can see would not have existed. If the neutron mass was 0.085% less than it is, the Universe would have been full of neutrons and nothing else.

If we cut the surface of a sphere up into faces, edges and vertices, and let F be the number of faces, E the number of edges and V the number of vertices, we will always get:

$$V - E + F = 2$$

**Fibonacci numbers** – 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144...

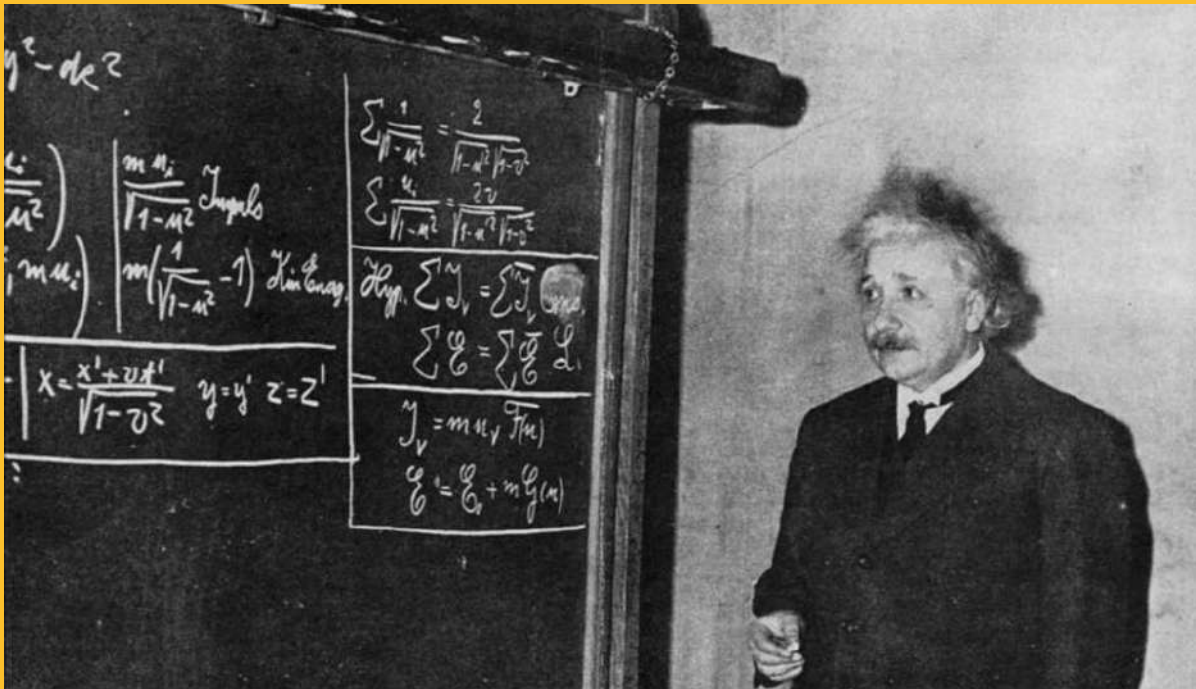
- Each number is the sum of the previous two.
- The ratio between the numbers = 1.618034 (**golden ratio**).

**From pinecones to the Hurricane Sandy, **Fibonacci** reflects various patterns found in nature.**

**The paths of anything you throw have the same shape, called an **upside-down parabola**.**

- All material particles have properties such as charge and spin.
- Space itself has properties such as dimensions.

These properties are purely mathematical.



*Einstein deriving special relativity, for an audience, in 1934.*

Equations aren't the only hints of mathematics that are built into nature: **there are also numbers involving not only motion and gravity**, but also areas as disparate as classical physics, quantum mechanics, and astronomy.

**Strong force** → Force that is responsible for binding together the fundamental particles of matter to form larger particles.

- **If stronger:** No hydrogen would have formed; atomic nuclei for most life-essential elements would have been unstable; thus, there would have been no life chemistry.
- **If weaker:** No elements heavier than hydrogen would have formed: again, no life chemistry.

**Weak force** → Force that is responsible for the radioactive decay of atoms.

- **If stronger:** Too much hydrogen would have been converted to helium in the big bang; hence, stars would have converted too much matter into heavy elements making life chemistry impossible.
- **If weaker:** Too little helium would have been produced from big bang; hence, stars would have converted too little matter into heavy elements making life chemistry impossible.

**Electromagnetic force** → Force that is responsible for most of the interactions we see in our environment today.

- **If stronger:** Chemical bonding would have been disrupted; elements more massive than boron would have been unstable to fission.
- **If weaker:** Chemical bonding would have been insufficient for life chemistry.

**c = 299,792,458 meters per second** – serves as the single limiting velocity in the universe, being an upper bound to the propagation speed of signals and to the speeds of all material particles.

#### **Ratio of electromagnetic force to gravitational force**

- **If larger:** All stars would have been at least 40% more massive than the sun; hence, stellar burning would have been too brief and too uneven for life support.
- **If smaller:** All stars would have been at least 20% less massive than the sun, thus incapable of producing heavier elements.

#### **Ratio of electron to proton mass**

- **If larger or smaller:** Chemical bonding would have been insufficient for life chemistry.



### Mass of the neutrino

- **If smaller:** Galaxy clusters, galaxies, and stars would have not formed.
- **If greater:** Galaxy clusters and galaxies would have been too dense.

### Ratio of exotic matter to ordinary matter

- **If larger:** The universe would have collapsed before the formation of solar-type stars.
- **If smaller:** No galaxies would have formed.

### Number of effective dimensions in the early universe

- **If larger or smaller:** Quantum mechanics, gravity, and relativity could not have coexisted; thus, life would have been impossible.

### Entropy level of the universe

- **If larger:** Stars would have not formed within proto-galaxies.
- **If smaller:** No proto-galaxies would have formed.

### Polarity of the water molecule

- **If greater:** Heat of fusion and vaporization would have been too high for life.
- **If smaller:** Heat of fusion and vaporization would have been too low for life; liquid water would not have worked as a solvent for life chemistry; ice would not have floated, and a runaway freeze-up would have resulted.

$$F_{\text{electrical}} = \frac{q_1 q_2}{4\pi\epsilon_0 r^2}$$

The electrical force decreases with increasing distance between the charged particles; when the distance is doubled, the force falls by a factor of 4.

The **greater the distance**  $d$  to the galaxy, the **higher the velocity**  $v$  with which it receded from us, according to the formula:

$$v = \text{Hubble parameter} \times d$$

The momentum  $p$  of something with **intrinsic mass**  $m_0$  moving with velocity  $v$  is simply given by:  $p = m_0v$  as long as  $v \ll c$ .

$$\beta = \frac{v}{c} \quad \text{Lorentz factor} = \frac{1}{\sqrt{1-\beta^2}}$$

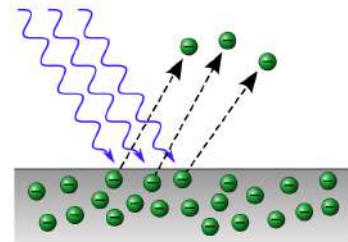
At  $v = 0.42c$ , **Lorentz factor** = 1.10 – which means the **effects of Einsteinian relativity** become noticeable.

Energy of the photon = Work Function of the surface + Kinetic energy of the emitted electron

$$h\nu = W + \frac{m_0v^2}{2}$$

If  $h\nu < W$ :

**No photoelectric emission**



The **Sagnac effect** occurs to a pair of light beams travelling in a circuit in the opposite direction. If the circuit is rotating, the beams will complete the circuit in different duration.

The effect was first demonstrated by **Georges Sagnac** in 1913.

$$m = \frac{m_0}{\sqrt{1-\beta^2}} \quad L = L_0\sqrt{1-\beta^2} \quad \Delta t = \frac{\Delta t_0}{\sqrt{1-\beta^2}}$$

If  $v = c$ :

$$m \rightarrow \infty$$

$$L \rightarrow 0$$

$$\Delta t \rightarrow \infty$$

Neutron  $\leftrightarrow$  proton + electron + antineutrino (**beta decay**)

Proton + electron  $\leftrightarrow$  neutrino + neutron (**electron capture**)

Proton + antineutrino  $\leftrightarrow$  positron + neutron (**inverse beta decay**)

- Closed Universe  $\rightarrow$  positively curved
- Open Universe  $\rightarrow$  negatively curved
- Flat Universe  $\rightarrow$  uncurved

$$\Delta x \cdot \Delta p \geq \frac{\hbar}{2}$$

The momentum and the position of a particle cannot be simultaneously measured with unlimited precision.

**The more precisely we determine a particle's position, the less precisely we determine its momentum.**

- Endothermic → absorbs heat
- Exothermic → releases heat

$$\text{Wavelength} = \frac{\text{Wave velocity}}{\text{Frequency}}$$



Wavelength of a wave with a constant velocity decreases with increase in frequency.

$$\frac{dA}{dt} = \frac{L}{2m} = \text{constant}$$

The areal velocity of a planet revolving around the sun in elliptical orbit remains constant which implies one-half its angular momentum divided by its mass remains constant. A consequence of this is that the Planet sweeps out equal areas in equal times.

$$b = \frac{L}{4\pi d^2}$$

A simple relation between

- The distance  $d$  to the star
- The apparent brightness  $b$  of the star and
- The luminosity (intrinsic brightness)  $L$  of the star.



At a particular Luminosity, the more distant a star is, the fainter its apparent brightness becomes as the square of the distance "d".

**Inverse Square Law of Brightness**

**Black hole temperature:**

$$T_{\text{BH}} = \frac{\hbar c^3}{8\pi k_B G M}$$

$$T_{\text{BH}} \propto \frac{1}{M}$$



- Higher mass Black Holes are colder and consequently radiate less.
- Smaller mass Black Holes are hotter and consequently radiate more.

$$T_{\text{BH}} = \frac{\hbar g}{2\pi c k_{\text{B}}} \longrightarrow T_{\text{BH}} \propto g$$



Temperature of the black hole increases with the surface gravity at the event horizon

The time that the black hole takes to dissipate is:

$$t_{\text{ev}} = \frac{5120\pi G^2 M^3}{\hbar c^4}$$

Thus, a black hole of one solar mass ( $M_{\text{sun}} = 2.0 \times 10^{30}$  kg) takes more than  $10^{67}$  years to evaporate — much longer than the current age of the universe at  $14 \times 10^9$  years.

$$t_{\text{ev}} \propto M^3$$



The evaporation time of the black hole increases with the cube of the mass of the black hole

$$t_{\text{ev}} = \frac{480c^2V}{\hbar c} \longrightarrow t_{\text{ev}} \propto V$$

The larger the Schwarzschild volume of the black hole, the less energy it gives off and the slower it evaporates

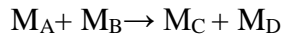
The total power radiated by the black hole is:

$$P = \frac{\hbar c^6}{15360\pi G^2 M^2}$$

$$P \propto \frac{1}{M^2}$$

The rate of loss of energy of the black hole decreases with the square of the black hole mass  
 (As black holes lose mass due to Hawking radiation, the rate of evaporation increases)

**In a nuclear reaction:**



$$Q = (M_A + M_B - M_C - M_D) c^2$$

The amount of energy released or absorbed in a nuclear reaction is called the **Q value**, or **reaction energy**.

- If  $M_A + M_B > M_C + M_D \rightarrow Q > 0 \equiv$  **exoergic reaction**
- If  $M_A + M_B < M_C + M_D \rightarrow Q < 0 \equiv$  **endoergic reaction**

The nuclear reaction that involve liberation of energy

The nuclear reaction that involve absorption of energy

An endoergic reaction will not proceed unless the incoming particle provides the reaction energy "Q".

$$L = \sigma T^4 A$$

- $L$  = luminosity
- $\sigma$  = Stefan-Boltzmann constant
- $A$  = surface area
- $T$  = temperature in Kelvin

If electrons were bosons, rather than fermions, then they would not obey the **Pauli Exclusion Principle**. There would be no life chemistry.

**A consequence of this is that:**

- The larger a star is, the more energy it puts out, and the more luminous it is.
- The star with a higher temperature will be more luminous than the star with lower temperatures.

If  $A$  stands for the surface area of a black hole (area of the event horizon), then the black hole entropy is given by:

$$S_B = \frac{k_B A}{4L_{\text{Planck}}^2}$$

A measure of black-hole internal information that is inaccessible to outside observers

The larger a black hole is, the more entropy it possess.

$$S_B \propto A, \text{ where } A \propto M^2$$

Black hole entropy is proportional to the surface area of its event horizon – which in turn is proportional to the square of the black hole mass.

## Wien's Law:

The peak wavelength is inversely proportional to its temperature in Kelvin.

{The wavelength of peak emission is inversely proportional to the temperature of the emitting object}



$$\lambda_{\max} = \frac{b}{T}$$

The amount of energy (radiation) released increases with temperature while the wavelength of peak emission falls

b is a constant of proportionality called **Wien's displacement constant**, equal to  $2.897771955... \times 10^{-3}$  mK

Thus, hotter objects emit most of their radiation at shorter wavelengths; hence they will appear to be bluer.

## Stellar Radiation Pressure:

$$P_{\text{radiation}} = \frac{4\sigma T^4}{3c}$$

Only a very small increase in temperature will result in a very large increase in the radiation pressure

Thus, a doubling of temperature means an increase of radiation pressure by a factor of 16.

The nuclear radius R can be approximated by the following formula:

$$R = r_0 \sqrt[3]{A}$$

A = Atomic mass number (the number of protons plus the number of neutrons) and  $r_0 = 1.25 \text{ fm} = 1.25 \times 10^{-15} \text{ m}$ .

Thus, size of nucleus depends on the mass number of nucleus.



## Eddington limit:

$$L_{\text{Edd}} = \frac{4\pi GMm_p c}{\sigma_T}$$

$\sigma_T$  is the Thomson scattering cross-section for the electron and  $m_p$  is the mass of a proton.

If a star exceeds this limit, its luminosity would be so high that it would blow off the outer layers of the star.

$$\text{Kinetic Energy} = \frac{1}{2} \times \text{mass} \times (\text{velocity})^2$$



Things with more mass can have more kinetic energy even though they are moving more slowly and things moving at much greater velocities can have more kinetic energy even if they have less mass.

The energy possessed by a body by virtue of its position

$$\text{Potential Energy} = \text{mass} \times \text{acceleration due to gravity} \times \text{height}$$



Potential Energy possessed by an object is directly proportional to its height above the surface of the earth. Therefore, it will increase in proportion to an increase in height – the potential energy will double with a doubling of height. A tripling of the height will result in a tripling of the potential energy.

### Viral Theorem for star:

$$\text{Thermal energy} + \text{gravitational potential energy} = \frac{1}{2} \text{gravitational potential energy}$$

$$\text{Thermal energy} = -\frac{1}{2} \text{gravitational potential energy}$$

$$K = -\frac{1}{2} U$$

As a consequence of this is that:

The thermal energy increases if the gravitational potential energy becomes more negative.

- Wavelength of light  $\ll$  size of particle : **Geometrical scattering**
- Wavelength of light  $\sim$  size of particle : **Mie scattering**
- Wavelength of light  $\gg$  size of particle : **Rayleigh scattering**

$k_B T \ll KE_{\text{Fermi}}$  : the electron gas is fully degenerate

$k_B T \sim KE_{\text{Fermi}}$  : the electron gas is partially degenerate

$k_B T \gg KE_{\text{Fermi}}$  : the electron gas is non-degenerate

The spin of the neutron, proton and electron are all  $\frac{1}{2}$ . If beta decay involves just a neutron becoming a proton and an electron, spin is not conserved.

Neutron  $\rightarrow$  proton + electron

Half integral  $\rightarrow$  integral

$$\frac{1}{2} \rightarrow \frac{1}{2} + \frac{1}{2}$$

Hence, the above reaction cannot take place if spin is to be conserved.

The electrostatic repulsion between two protons is  $\frac{e^2}{4\pi\epsilon_0 r^2}$  while the gravitational attraction between them is  $\frac{Gm_p^2}{r^2}$ . The ratio of these two forces is  $\frac{e^2}{4\pi\epsilon_0 Gm_p^2}$ . This expression is independent of distance between them, so the relative strength of the forces is the same throughout all space.

If

$$\frac{mv^2}{2} > \frac{GMm}{r}$$

Object of mass  $m$  will escape the gravitational field of mass  $M$ .

In classical physics, it is possible to exactly specify both position and momentum simultaneously.

**Quantum mechanics:**

If we try to localize a particle spatially, we lose information about its momentum.

A light year is the distance traveled by light in a year:

$$1 \text{ light year} = (\text{speed of light}) \times (1 \text{ year}) = 3 \times 10^{10} \text{ cms}^{-1} \times 3 \times 10^7 \text{ s} = 9 \times 10^{17} \text{ cm.}$$

- Water freezes at 273 K ( $\equiv 0^\circ\text{C}$ )
- Water boils at 373 K ( $\equiv 100^\circ\text{C}$ )

**Intrinsic energy of proton = KE of quarks+ PE of quarks + intrinsic energy of quarks**

Any reaction that can take place in nature must follow this rule, no matter what force is responsible for the reaction:



- In any reaction the total charge of all the particles entering the reaction must be the same as the total charge of all the particles after the reaction.

Because  $E = mc^2$  (the equation that represents the correlation of energy to matter: essentially, energy and matter are but two different forms of the same thing) and due to the fuzziness of quantum theory (that implies: photon carries mass proportional to its frequency i.e.,  $h\nu = mc^2$ ), some of the most incredible mysteries of the quantum realm (a jitter in the amorphous haze of the subatomic world) get far less attention than Schrödinger's famous cat. Virtual particle-antiparticle pairs of mass ( $\Delta m = \frac{h\Delta\nu}{c^2}$ ) are continually created out of energy  $\Delta E$  of the empty space consistent with the Heisenberg's uncertainty principle of quantum mechanics (which implies:

$$\Delta mc^2 \times \Delta t \geq \frac{\hbar}{2}$$

where:  $\Delta t$  stands for time during which virtual particle-antiparticle pairs of energy ( $\Delta mc^2 = h\Delta\nu$ ) appear together, move apart, then come together and annihilate each other giving energy back to the space without violating the law of energy conservation (which states that energy can neither be created nor destroyed; rather, it can only be transformed from one form to another).

Spontaneous births and deaths of roiling frenzy of particles so called virtual matter – antimatter pairs momentarily occur everywhere, all the time – violate the Energy-momentum relationship:  $E = \sqrt{m_0^2 c^4 + p^2 c^2}$  – is the conclusion that mass and energy are interconvertible; they are two different forms of the same thing. The word virtual particles literally mean that these particles are not observed directly, but their indirect effects are measured to a remarkable degree of accuracy. Their properties and consequences are well established and well understood consequences of quantum mechanics.

**Hubble's law**

Consequence of the expansion of the space through which light is travelling.

$$\frac{m_{\text{proton}}c^2}{k_B} = T \quad \left. \vphantom{\frac{m_{\text{proton}}c^2}{k_B}} \right\} \text{Temperature below which proton is effectively removed from the universe}$$

For particles moving at speeds very close to that of light:

$$E = \sqrt{m_0^2c^4 + p^2c^2}$$

$pc \gg m_0c^2$ :

$$E = pc$$

Hence the matter behaves similarly to the radiation.

- The angles in a triangle when added together sum up to  $180^\circ$ .
- The circumference of a circle divided by its diameter is a fixed number called  $\pi$ .
- In a right angled triangle the lengths of the sides are related by  $c = \sqrt{a^2 + b^2}$  where  $c$  is the length of the side opposite to the right angle.

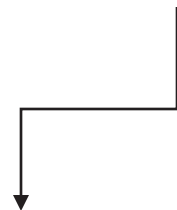
- $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$
- $1 \text{ keV}$  is a thousand eV
- $1 \text{ MeV}$  is a million eV
- $1 \text{ GeV}$  is a thousand million eV
- $1 \text{ TeV}$  is a million million eV

Particles can only spin at a rate that is a multiple of  $\frac{h}{2\pi}$ .

- Fermions (**quarks and leptons**) spin at  $\frac{1}{2} \times \frac{h}{2\pi}$ .
- Bosons (**photons and gluons**) spin at  $1 \times \frac{h}{2\pi}$  or  $2 \times \frac{h}{2\pi}$ .

**Euler's formula:**

$$e^{\pi i} + 1 = 0$$



Connects the five fundamental constants of mathematics ( $e, \pi, i, 0, 1$ ).

[Imaginary number  $i = \sqrt{-1}$ ].

|                      |                                  |
|----------------------|----------------------------------|
| Maxwell equations    | electromagnetism                 |
| Schrödinger equation | quantum mechanics                |
| Balmer equation      | Interpretation of atomic spectra |
| Yang-Mills equation  | SU(2) gauge symmetry of isospin  |
| Dirac equation       | relativistic quantum mechanics   |
| Higgs field equation | symmetry breaking                |
| Einstein equations   | relativity                       |
| The logistic map     | chaotic dynamics                 |

### Noether's Theorem (1918):

"For every continuous symmetry there is a corresponding conserved quantity [such as electric charge] and vice versa."

"Even if there is only one possible unified theory, it is just a set of rules and equations...  
What is it that breathes fire into the equations and makes a universe for them to describe?"

"Why does the universe go to the bother of existing?"

– **Stephen Hawking**

$$(i \gamma_{\mu} d^{\mu} - m) \psi = 0$$

**The Dirac Equation that predicts the existence of antimatter**

where:

- $i$  = imaginary number
- $\gamma_{\mu}$  = Pauli matrices
- $d^{\mu}$  = derivative in 4 dimensions
- $m$  = fermion mass
- $\psi$  = wave function



**Bayes' Theorem:**

$$P(H | E) = \frac{P(E|H) \cdot P(H)}{P(E)}$$

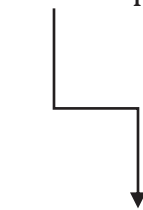
H represents a hypothesis and E the evidence.

- P (H | E) – the probability of H given E is true
- P (E | H) – the probability of E given H is true
- P (E) – the probability of E
- P (H) – the probability of H

The number 0 is the neutral element of addition:

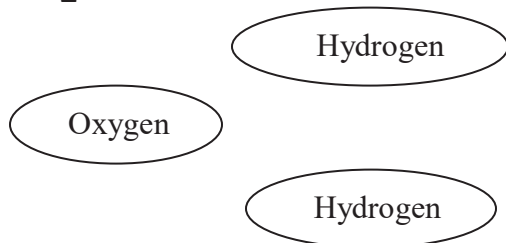
$$\left. \begin{array}{l} 1 + 0 = 1 \\ 23 + 0 = 23 \end{array} \right\} \begin{array}{l} \text{Adding 0} \\ \text{Nothing happens} \end{array}$$

$$F = G \times \frac{m_1 m_2}{r^2}$$



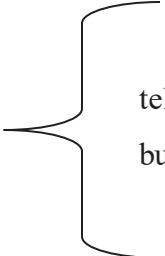
Constant that controls the strength of gravity

H<sub>2</sub>O



Consisting of one oxygen atom and two hydrogen atoms, water molecule plays a special role in the chemistry of life.

The  $\Delta^{++}$  is more massive than the sum of the masses of the proton and the pion ( $\pi^+$ ). This means that it is energetically possible for the  $\Delta^{++}$  to decay into a proton and a pion.

**General relativity**  tell us about the geometry of spacetime,  
but not the topology.

The Planck mass  $\sqrt{\frac{\hbar c}{G}}$  is roughly 24,000,000,000,000,000,000 ( $2.4 \times 10^{22}$ ) times the mass of the electron.

Observations



Hypothesis



Experiment



Laws

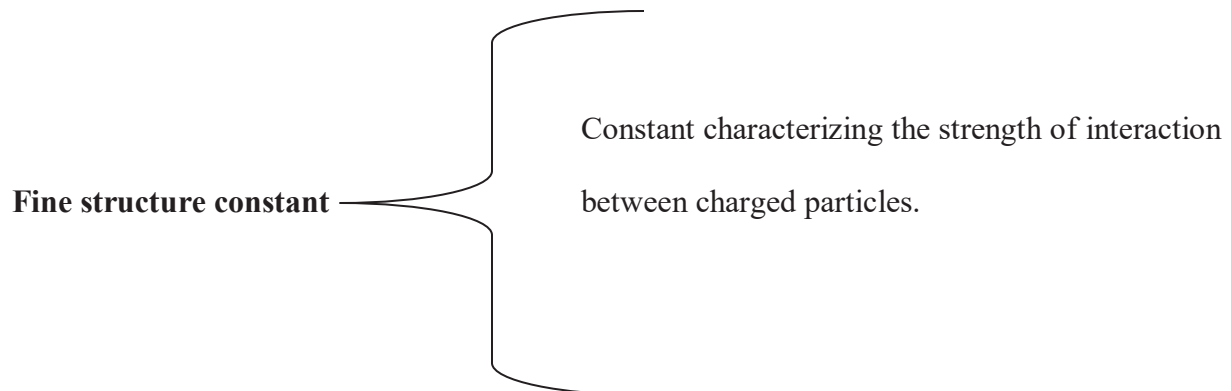


Theory

- Planck's law is accurate at all wavelengths.
- Wien's Law is a good approximation at short wavelengths.
- The Rayleigh-Jeans Law is a good approximation at large wavelength.

**Five Equations That Changed the World:**

- $F = \frac{GMm}{d^2}$  (Newton's Law of Universal Gravitation)
- $P + \rho \times \frac{1}{2} v^2 = \text{constant}$  (Bernoulli's Law of Hydrodynamic Pressure)
- $\nabla \times E = -\frac{\partial B}{\partial t}$  (Faraday's Law of Induction)
- $E = mc^2$  (Albert Einstein's mass–energy equivalence)
- $S_{\text{universe}} > 0$  (Clausius's Law of Thermodynamics)



**Titius–Bode law:**

The distance from the Sun to the  $n^{\text{th}}$  planet is  $0.075 \times 2^n + 0.4$  astronomical units.

$$c = \frac{1}{\sqrt{\text{Vacuum permeability} \times \text{Vacuum permittivity}}}$$

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

Determined by the electromagnetic properties of  
free space –  $\mu_0$  and  $\epsilon_0$

Quantum mechanics + general theory of relativity → quantum theory of gravity

If the density perturbations were much weaker, then galaxies may never have coalesced. Without galaxies there would be no buildup of heavy elements, and it is unlikely that planets, and life, would have emerged.

Entropy of Universe = Entropy of visible Universe + Entropy of dark matter + Entropy of black holes

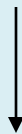
The total energy of the star = internal energy due to thermal motion and radiation + gravitational potential energy

- Stars with mass  $> 0.08 M_{\text{sun}}$  burn hydrogen.
- Stars with mass  $> 0.5 M_{\text{sun}}$  burn hydrogen and helium.
- Stars with mass in the range of 1 to  $8 M_{\text{sun}}$  continue nucleosynthesis up till the production of carbon.
- Stars with mass  $> 10 M_{\text{sun}}$  synthesize all the elements up to iron and nickel.

**General relativity:**  $\Omega$  determines the curvature ( $k$ ), or "shape", of the universe:

- $\Omega = 1$ : flat universe,  $k = 0$
- $\Omega > 1$ : closed (or bound) universe,  $k = +1$
- $\Omega < 1$ : open (or unbound) universe,  $k = -1$

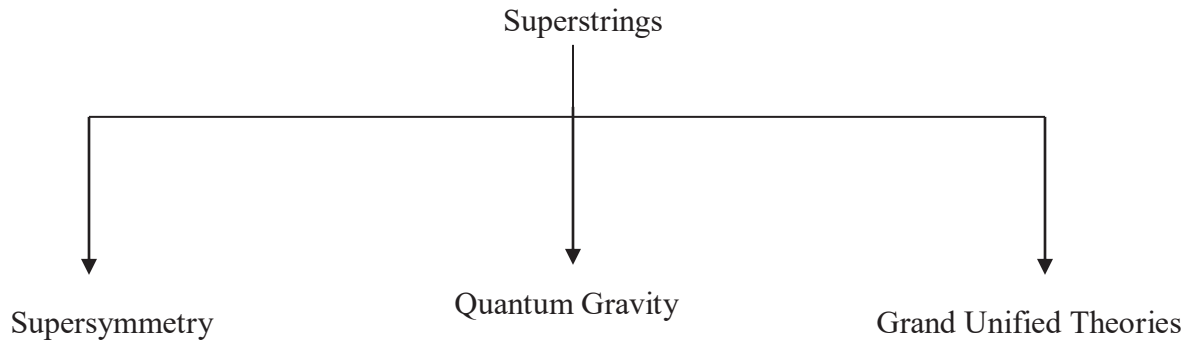
### Hydrostatic Equilibrium



**The outward radiation pressure precisely balances inward gravitational pressure during the Main Sequence Phase of a star's evolution. The star is stable because these two pressures are perfectly balanced (this means the star neither shrinks nor expands)**

Rate of energy production in the **pp-process** of hydrogen burning  $\propto (\text{Temperature})^4$

Rate of energy production in the **CNO-process** of hydrogen burning  $\propto (\text{Temperature})^{18}$



- Size of our universe  $\sim 10^{26}$  m
- The distance Earth–Sun is  $\sim 1.5 \times 10^{11}$  m
- The radius of the Sun is  $\sim 7 \times 10^8$  m
- The radius of the Earth is  $\sim 6.4 \times 10^6$  m
- Rocks, Humans, . . .  $\sim 1$  m
- Grains of sand  $\sim 10^{-3}$  m
- Viruses  $\sim 10^{-7}$  m
- Simple molecules  $\sim 10^{-9}$  m
- Atoms  $\sim 10^{-10}$  m

$$\text{Redshift} = \frac{\lambda_{\text{observed}}}{\lambda_{\text{emitted}}} - 1$$

always positive, i.e. observed radiation is redder than the emitted one.

**Implies:**

Universe is in expansion

## Thermodynamics of the universe:

$$0 = dQ = dU + PdV$$

$$dU = - PdV$$

where  $Q$  is the total heat which is assumed to be constant,  $U$  is the internal energy of the matter and radiation in the universe,  $P$  is the pressure and  $V$  the volume

**Energy density:**  $u = \frac{U}{V}$

$$du = - (P + u) \frac{dV}{V} = - 3(P + u) \frac{da}{a}$$

For radiation,  $P = \frac{u}{3}$  whereas for matter  $P \ll u$  and the pressure can be neglected. Thus we get:

For radiation

$$du = - 4u \frac{da}{a} \text{ thus } u \text{ is proportional to } a^{-4}$$

For matter

$$du = - 3u \frac{da}{a} \text{ thus } u \text{ is proportional to } a^{-3}$$

- $a$  being the scale factor of the universe.

As the universe expands, the wavelength of light " $\lambda$ " is stretched linearly:

$$\lambda \propto a$$

which implies that photons lose energy as  $E \propto \frac{1}{a}$

$$\text{Temperature of relativistic particles} \propto \frac{1}{\text{scale factor of the universe}}$$

$$\text{Temperature of non-relativistic particles} \propto \frac{1}{(\text{scale factor of the universe})^2}$$

| Electromagnetic wave | Wavelength                       |
|----------------------|----------------------------------|
| Radio                | ~ 10 cm or larger                |
| Microwave            | ~ 1 cm                           |
| Infrared             | ~ 10 <sup>-3</sup> cm            |
| Visible              | ~ 10 <sup>-5</sup> cm            |
| Ultraviolet          | ~ 10 <sup>-6</sup> cm            |
| X-rays               | ~ 10 <sup>-8</sup> cm            |
| Gamma-rays           | ~ 10 <sup>-9</sup> cm or smaller |

### Three Classical Tests of General Relativity:

- Precession of Mercury's orbit
- Deflection of starlight (gravitational lensing)
- Gravitational Redshift

- **Quantum physics** → (Behavior of very small things)
- **Relativity theory** → (Behavior of very large things)

If all of the galaxies are redshifted, then they are all moving apart from each other!!



**(This is the evidence that our Universe is expanding)**

Because  $E = mc^2$ :

- A light bulb filament has more mass when it is energized with electricity than when it is turned off.
- A hot cup of tea has more mass than the same cup of tea when cold.



**Gravity will affect anything carrying energy**



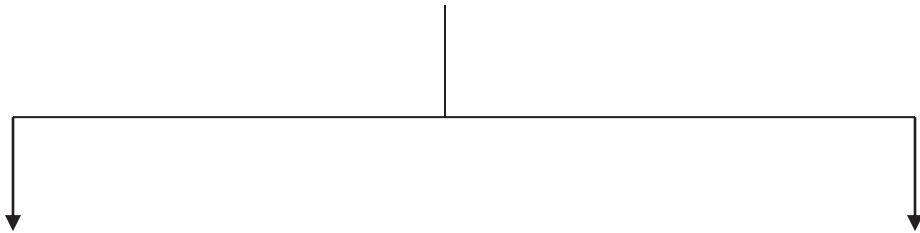
Root of the construction of Einstein's equations which describe gravity

Light intensity drops as  $\frac{1}{(\text{distance})^2}$

- In an open universe (**negative curvature**): the angles in a triangle add up to less than 180°.
- In a closed universe (**positive curvature**): the angles in a triangle add up to more than 180°.
- In a flat universe (**zero curvature**): the angles in a triangle add up to 180°.

The energy of the universe is constant.  
The entropy of the universe tends to a maximum.

**Theory**



**External Reality Theory (ERT)**

External reality exists completely independent of human beings

**Mathematical Universe Theory (MUT)**

External physical reality is a mathematical structure

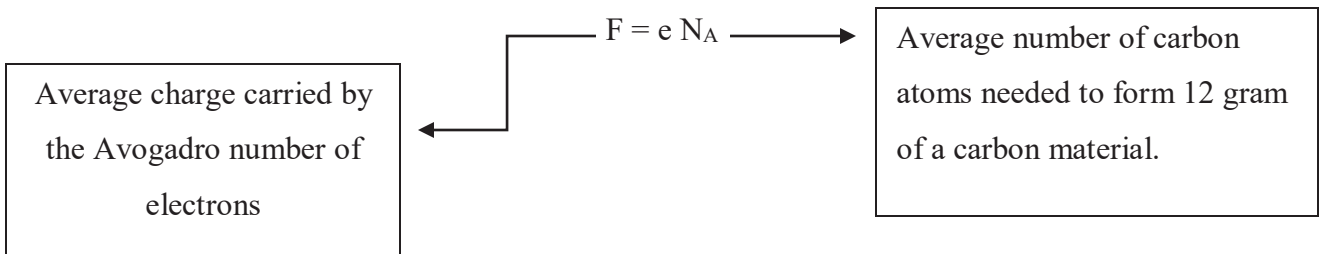


When associated with  $c$  and with the reduced Planck's constant  $\hbar$ , it leads to the definition of the Planck's time:

$$t_{\text{Planck}} = \sqrt{\frac{\hbar G}{c^5}} = 5.4 \times 10^{-44} \text{ seconds}$$

The shortest possible time interval that can be measured.

**Faraday constant:**



The nuclear charge  $Q$  can be approximated by the following formula:

$$Q = Ze$$

$Z$  = Atomic number (the number of protons).

Thus, charge of nucleus depends on the number of protons.

$$Q = \frac{\text{number of protons}}{N_A} \times F$$

$$Q = \text{number of moles of proton} \times F$$

**The strong coupling constant** defines the strength of the force that holds protons and neutrons together.

The Universe is made up of three things:

- Vacuum
- Matter
- Photons

**Total energy density of the universe:**

$$\rho = \rho_{\text{vacuum}} + \rho_{\text{matter}} + \rho_{\text{radiation}}$$

$\rho_{\text{vacuum}} = \frac{\Lambda c^2}{8\pi G}$  is constant and independent of time. The cosmological constant " $\Lambda$ " has negative pressure equal and opposite to its energy density and so causes the expansion of the universe to accelerate.

**Matter:**

$R \rightarrow$  scale factor of the universe

$$E = mc^2$$

For  $N$  particles:

$$E = Nmc^2$$

$$\rho_{\text{matter}} \propto \frac{Nmc^2}{R^3}$$

**Radiation:**

$$E = \frac{hc}{\lambda} \quad \text{and} \quad \lambda \propto R$$

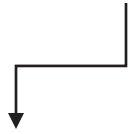
For  $N$  photons:

$$E \propto \frac{Nhc}{R}$$

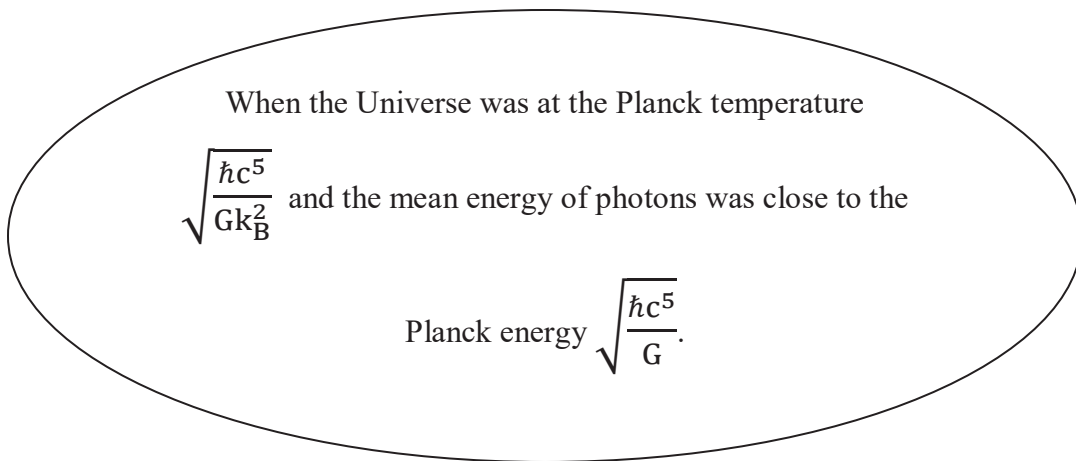
$$\rho_{\text{radiation}} \propto \frac{Nhc}{R^4}$$

- $\rho_{\text{vacuum}} \propto R^0$
- $\rho_{\text{matter}} \propto R^{-3}$
- $\rho_{\text{radiation}} \propto R^{-4}$

$$\frac{m_{proton}}{m_{electron}} = 1836.15267389$$



Changing their values changes the physical phenomena



If the thermal kinetic energy of stellar gas cloud wins over gravity:

$$\frac{3}{2} N k_B T > \frac{3GM^2}{5R} \rightarrow \text{Expansion}$$

If

$$\frac{3GM^2}{5R} > \frac{3}{2} N k_B T \rightarrow \text{gravity wins: collapse!}$$

Solar mass =  $2 \times 10^{30}$  kg – of which about 70% is hydrogen, 28% helium, and 2% consists of other elements. Only about a seventh part of that hydrogen mass is available at any time for hydrogen fusion in the core of the Sun.

Because relativistic mass =  $\frac{\text{rest mass}}{\sqrt{1-\frac{v^2}{c^2}}}$ : the faster an particle moves, the more kinetic energy it

possess. But according to  $E= mc^2$ , kinetic energy adds to an particle's mass, so the faster an particle moves, the harder it is to further increase the particle's speed. This effect is really significant only for particles moving at speeds close to the speed of light. As a particle approaches the speed of light, its mass increases ever more quickly to infinite, so it take infinite amount of energy to speed it up further. This is the reason that any material particle is forever confined by relativity to move at speeds slower than the speed of light. Only photons that have no intrinsic mass move at the speed of light.

$$E^2 = p^2c^2 + m_0^2c^4$$

$m_0 = 0$ :

$$E = pc$$

The **Compton wavelength of a particle** characterizes the length scale at which the wave property of a given particle starts to show up. In an interaction that is characterized by a length scale larger than the Compton wavelength, particle behaves classically (i.e., no observation of wave nature). For interactions that occur at a length scale comparable than the Compton wavelength, the wave nature of the particle begins to take over from classical physics.

At Planck length ( $\sqrt{\frac{\hbar G}{c^3}}$ ), the gravitational force is as strong as the other forces and space-time is "foamy" – filled with tiny bubbles and wormholes appearing and disappearing into the vacuum.

**Rayleigh scattering law:** The amount of scattering of light is inversely proportional to the fourth power of the wavelength.

$$I \propto \frac{1}{\lambda^4}$$

Thus, Rayleigh scattering is more intense at shorter wavelengths.

**Supersymmetry** — The positive zero point energy of the boson field exactly cancels the negative zero point energy of the fermion field.

$h \rightarrow 6.62607004 \times 10^{-34} \text{ m}^2 \text{ kg/s}$  — Because  $h$  is too small: **Quantum mechanics is for little things.**

**Einstein's Relativity:**

Motion and Gravity make Clocks tick slowly.

**Gravity** pulls everything in, but a mysterious force called **dark energy** tries to push it all back together again. Our fate relies on which force will win the desire to succeed.

Because of

**CP violation** (violation of charge conjugation parity symmetry)

there was more matter than antimatter right after the Big Bang.

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Tachyons (if they exist) have  $v > c$ . This means that  $m$  is imaginary!

**General theory of relativity** describes gravity, ignoring quantum mechanics.

**Math in Nature:**

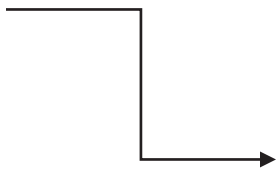


|                    |   |
|--------------------|---|
| Hexagon            | Bee Hive  |
| Concentric Circles | Ripples of a pond when a stone hits the surface of the water. |

**Doppler Effect:**

- When a wave source moves toward an observer, its waves appear to have a shorter wavelength.
- When a wave source moves away from an observer, its waves appear to have a longer wavelength.

In more than three space dimensions, planetary orbits would be unstable and planets would either fall into the sun or escape its attraction altogether

What goes up must get down  **Newton's law of gravity**

What goes up need not descend– if it is shot upward faster than the **escape velocity**  $\left(\sqrt{\frac{2GM}{R}}\right)$

Since gravity weakens with distance, the earth pulls on your head with less force than it pulls on your feet, which are a meter or two closer to the earth's center. The difference is so tiny we cannot feel it, but an astronaut near the surface of a black hole would be literally torn apart.

Because:

$$2\pi r = n\lambda$$

Only orbits with circumferences corresponding to a whole number of electron wavelengths could survive without destructive interference.

Because:

$$r = \frac{3GM}{c^2}$$

The photon spheres can only exist in the space surrounding an extremely compact object (a black hole or possibly an "ultracompact" neutron star).

- **In phase** → wave crests and troughs reinforce each other.
- **Out of phase** → wave crests and troughs cancel out.

- The energy above which (**Grand unification energy**  $\approx 10^{16}\text{GeV}$ ), the electro-magnetic force, weak force, and strong force become indistinguishable from each other.
- Since the graviton has no mass of its own, the **gravitational force of attraction** between the sun and every planet is long range.
- The proton and neutron masses are so similar; they differ only by the replacement of an up quark with a down quark.

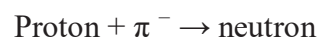
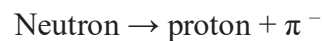
Because:

$$\frac{E}{B} = c$$

Electric and magnetic fields turn into each other in a wavelike motion, creating an electromagnetic field that travels at the speed of light.

When **two black holes** collide, they merge, and the area of the final black hole is greater than the sum of the areas of the original holes.

Inside the nucleus of an atom, a proton is never permanent a proton and a neutron is never permanently a neutron. They keep on changing into each other. A neutron emits a pi meson and become proton and a proton absorbs a pi meson and become a neutron.



There is no escape from a black hole in **classical theory**, but quantum theory enables energy and information to escape.

The **inherent goal of unification** is to show that all of these forces are, in fact, manifestations of a single force. We can't perceive this unity at the low energies of our everyday lives, or even in our most powerful accelerators at CERN. But close to the Big Bang temperatures, at inconceivably high energies... If the forces unify, the proton can be unstable, and eventually decay ...

Proton  $\rightarrow$  positron + neutral pion

Neutral pion  $\rightarrow$  2 gamma ray photons

- Accelerated massive bodies give off gravitational waves just as bound electrons in an atom emit electromagnetic radiation
- A rotating neutron star (a tiny, burnt out star) generates regular pulses of radio waves.
- Quantum mechanics says that the position of a particle is uncertain, and therefore that there is some possibility that a particle will be within an energy barrier rather than outside of it. The process of moving from outside to inside without traversing the distance between is known as **quantum tunneling**, and it is very important for the fusion reactions in stars like the Sun.

According to string theory, our universe is made up of small vibrating strings whose size is  $10^{23}$  times smaller than protons. A **String** can do something aside from moving – it can vibrate in different ways. One mode of vibration makes the string appear as a photon, another as a neutron, and so on...

Because

$$dM = \frac{k}{8\pi} dA + \Omega dJ + \Phi dQ$$

M stands for mass, k for surface gravity, A for area of the event Horizon, J for angular momentum,  $\Omega$  for angular velocity, Q for charge and  $\Phi$  for the electrostatic potential

the size and shape of the black hole depends only on its mass, charge and rate of rotation, and not on the nature of the star that had collapsed to form it.

**The electromagnetic and weak interactions lose their symmetry below 100 GeV**

Entropy (a thermodynamic measure of untidiness in a system and a measure of how much information a system contains) is defined as:

$$S = k_B \ln \{\text{number of states}\}$$

which, for N particles of the same type, is

$$S = k_B \ln \{(\text{no of one-particle states})^N\}$$

$$S = k_B N \ln \{ \text{a not-too-big number} \}$$

$$S = k_B N$$

This means: the more particles, the more disorder.

**Hund's rule:** every orbital in a subshell is singly occupied with one electron before any one orbital is doubly occupied, and all electrons in singly occupied orbitals have the same spin.

Because:

$$\text{Kinetic energy of the emitted electron} = \text{photon energy} - 13.6 \text{ eV}$$

photons need an energy  $> 13.6 \text{ eV}$  to ionize hydrogen atom.

$$11 \times 11 \rightarrow 121$$

$$111,111,111 \times 111,111,111 \rightarrow 12,345,678,987,654,321$$

Palindrome number – a number that reads the same forwards or backwards

If particle A enters the ergosphere of a Kerr black hole, then it splits into particles B and C.

$$E_A = E_B + E_C$$

Particle C with Energy  $E_C < 0$  (negative energy)  $\rightarrow$  falls into the black hole.

Particle B with Energy  $E_B > E_A \rightarrow$  escapes

The added negative energy particle will slow down the spinning of the Kerr black hole and reduce its energy and therefore its mass.

**Waves over time**  $\rightarrow$  As the waves decay, their amplitude becomes smaller and, eventually, the waves disappear

Because:

$$\nabla \times \vec{E} = - \frac{d\vec{B}}{dt}$$

Electricity and magnetism are related.

"This difficulty [the inconsistency of the laws of gravity with a finite mean density of matter] also arises in the general theory of relativity. However, I have shown that this can be overcome through the introduction of the so-called “ $\lambda$ -term” to the field equations... I showed that these equations can be satisfied by a spherical space of constant radius over time, in which matter has a density  $\rho$  that is constant over space and time."

– **Albert Einstein**

**German astronomer** and mathematician **Johannes Kepler** who came from a poor Protestant family in Germany – established that the planet orbit the sun in a elliptical path, meaning that the planet to Sun distance is constantly changing as the planet goes around its orbit. Kepler's laws of motion tell how the planet moves on this way. Later **Isaac Newton's universal law of gravitation** explained why the planets move in this way.

**Tycho's model**

Planets orbit around the Sun and the Sun orbit around the Earth at the center of the Universe.

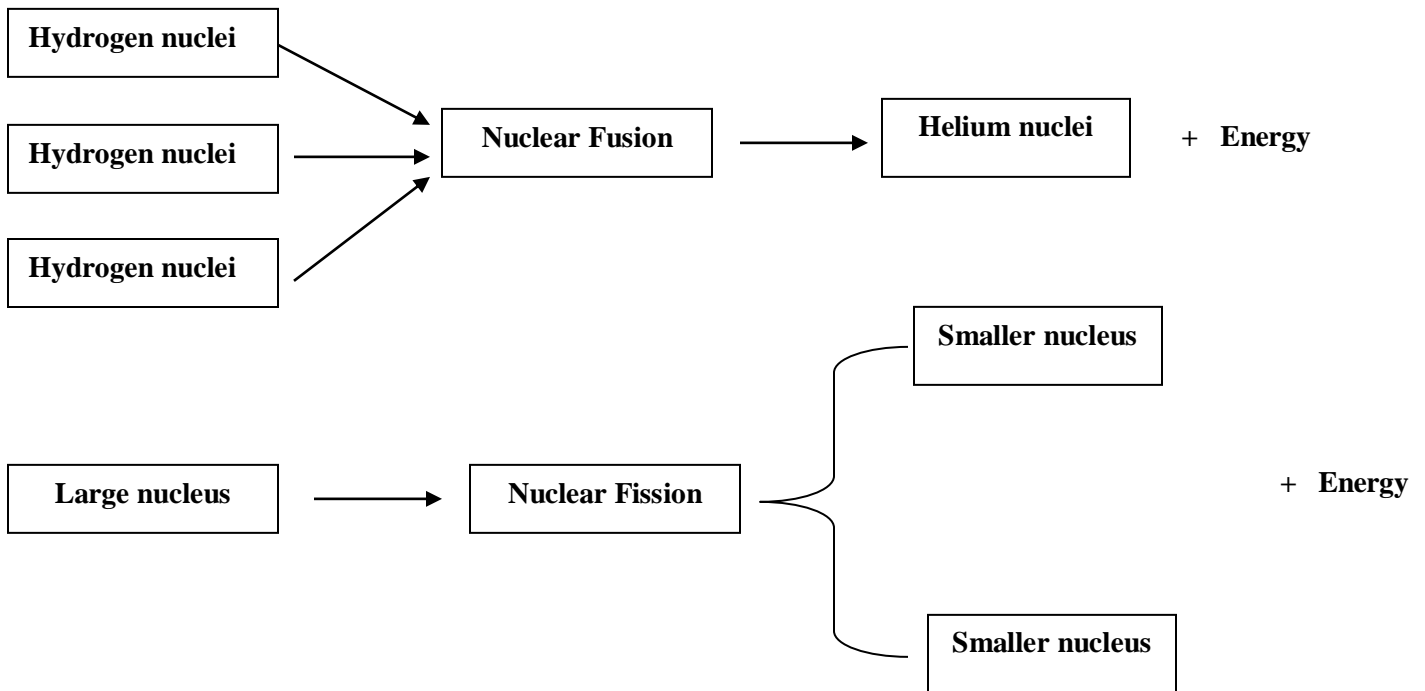
**Electromagnetic wave**

The undulating strength of the electric and magnetic disturbance – propagating through space – carrying electromagnetic radiant energy

The **expansion of the Universe** can be compared to the expanding surface of a balloon that is being inflated. As more air is blown into it, we would see the surface area of the balloon expanding and every point on its surface getting further and further away from one other.



- In a bound atom of hydrogen the negatively charged electron moves round the positively charged nuclei. In high temperature plasma the nuclei and electrons are no longer bound.
- Motion of stars in galaxies reveals the existence of hypothetical form of mass thought to account for approximately 85% of the mass in the universe and about 27% of its total mass–energy density, or "**dark matter**," whose nature remains unknown.

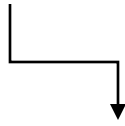


**Atomism**

The world view that everything is built up from two fundamental principles: atom (**fundamental indivisible component**) and void.

**The sum of multiple waves** → superposition  
**(The resulting wave form is stable in time and space)**

## Complementarity Principle



Wave and particle or **position and momentum** cannot be observed at the same time.

## Ontology:

What the underlying structure of reality is?

**Paradigm** → framework for thinking about the nature of reality

So far as we know, all the fundamental laws of physics, such as Newton's equations, are reversible. Then where does irreversibility come from? It comes from order going to disorder, but we do not understand this until we know the origin of the order. Why is it that the situations we find ourselves in every day are always out of equilibrium?

- **Richard Feynman**

|  |   |
|--|---|
| <b>Aristotle</b> (384–322 B.C)         | The earth is spherical in shape   |
| <b>Aristarchus</b> (312-230 B.C)       | The Universe is Sun-centered  |
| <b>Johannes Kepler</b> (1571–1630)     | Planets move around the Sun in Orbits which are not circular but elliptical |
| <b>Nicolaus Copernicus</b> (1473–1543) | The Sun is at the centre of the Solar System                                |
| <b>Galileo Galilei</b> (1564–1642)     | The Sun has both hot high temperature and dark low temperature spots        |

**When two numbers are added, their order is not important**

$$1 + 2 = 2 + 1$$

|                              |   |
|------------------------------|---|
| Arithmetic and number theory | <b>patterns of number and counting</b>    |
| Geometry                     | <b>patterns of shape</b>                  |
| Calculus                     | <b>patterns of motion</b>                 |
| Logic                        | <b>patterns of reasoning</b>              |
| Probability theory           | <b>patterns of chance</b>                 |
| Topology                     | <b>patterns of closeness and position</b> |

**Gravity and Distance:**

$$F_1 = \frac{GMm}{r^2}$$

If the distance between the masses triples, the gravitational force decreases by three squared, or nine:

$$F_3 \text{ (force at thrice the distance)} = \frac{GMm}{(3r)^2} = \frac{GMm}{9r^2} = \frac{1}{9} F_1$$

Increasing the distance by twenty times would decrease the gravitational attraction by four hundred times:

$$F_{20} \text{ (force at twenty times the distance)} = \frac{GMm}{(20r)^2} = \frac{GMm}{400r^2} = \frac{1}{400} F_1$$

Since the Moon's mass is  $7.35 \times 10^{22}$  kg i.e., about 1.2 percent of Earth's mass, it has a much weaker gravitational pull on us. This means our weight would be less on the Moon than on Earth. In fact, we would weigh about one-sixth what we weigh on Earth.

## Spontaneous generation theory



Different kinds of nonliving matter give rise to different kinds of living creatures

(Rotting meat gives rise to flies while old rags give rise to mice)

## Albert Einstein's theory:

The entire universe can expand or contract – just like the overall stretching or shrinking of an elastic sheet

## Max Tegmark's 4 distinct types of parallel universes:

- Parallel universes with the same laws of physics but different initial conditions
- Parallel universes with the same equations of physics but perhaps different constants of nature
- Parallel universes superimposed in the same physical space but mutually isolated and evolving independently
- Parallel universes with different mathematical structures

**365** → the number of days in a year



$$10^2 + 11^2 + 12^2$$

Sum of 3 consecutive squares

$$13^2 + 14^2$$

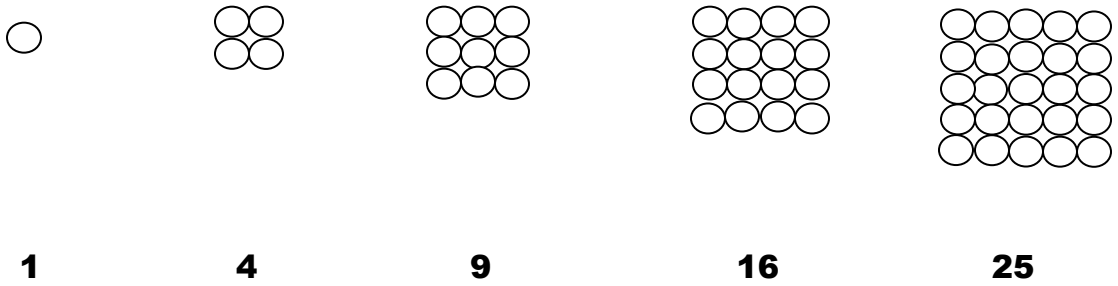
Sum of the next 2 squares

**28** → the number of days in the lunar month

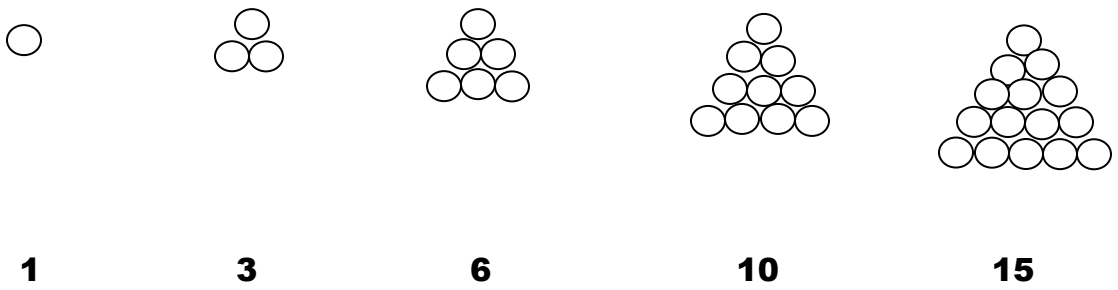


|                      |   |
|----------------------|---|
| $1 + 2 + 4 + 7 + 14$ | Sum of all of its divisors                    |
| $1^3 + 3^3$          | Sum of the cubes of the first two odd numbers |

**Square numbers:** Numbers that can be arranged in a square



**Triangular numbers:** Numbers that can be arranged in a triangle



**Palindromic numbers**



Numbers reading the same forwards and backwards

**121**  
**523323325**

Henceforth space by itself, and time by itself, are doomed to fade away into mere shadows, and only a kind of union of the two will preserve independence

**- Hermann Minkowski, 1908**

Object moves at constant velocity in an inertial frame  $\iff$  Object experiences zero net force

**In string theory:**

- Laws of physics
- Particle spectrum
- Nature of forces



Dictated by shape + size (**geometry**) of dimensions

A photon moving upwards in gravitational field is redshifted. Since

$$v = \frac{1}{T}$$

the photon's period gets longer. Observer 1 will measure a longer period than Observer 2.

- This gravitational time dilation effect is unnoticeable in our daily experience!
- This is tiny in Earth's gravitational field, but large in a black hole's.

**In one second, 4.5 million tons of rest mass is converted to radiant energy in the sun.**

A cluster of galaxies consists of 3 components:

- Galaxies
- Hot Gas
- Dark Matter

**Spatial dimensions  $\geq 4$ :**

Electrons fall on the nuclei and therefore the atomic structure of matter does not exist.

The atomic matter and therefore life are possible only in 3-dimensional space

- If the electron charge were increased by a factor  $\sim 3$  no nuclei with atomic number  $> 5$  would exist and no living organisms would be possible

So one reason why scientists do mathematical theories is because they surprise us, they become smarter than us, and eventually we become the students of the theory.

**- Donald Hoffman**

**Entropy change**  
 $\Delta S \geq 0$

- = 0 (reversible process)
- > 0 (irreversible process)

We are the cosmos made conscious and life is the means by which the universe understands itself...

**Brian Cox**

**Boltzmann factor:**  $e^{-\frac{E}{k_B T}}$

Express the probability of the system **occupying the state of** energy E at absolute temperature T

- Plasma → ions + electrons
- When electrons interact with the electric (Coulombic) fields of ions – electrons would be deflected and a portion of their kinetic energy would be lost.

Final KE of electrons = initial KE of electrons – energy of emitted X-ray photons

The 'lost' kinetic energy is emitted as X-ray photons (**Bremsstrahlung radiation**)

The KE per electron is  $\frac{3 k_B T}{2}$ . When an electron gives up all its kinetic energy, then the **Bremsstrahlung** photon is emitted with highest frequency  $\nu$ :

$$\nu = \frac{3 k_B T}{2h}$$

$$\lambda T = \frac{2c_2}{3} \text{ (where } c_2 \text{ is the second radiation constant)}$$

Since boat displaces a weight of water equal to its own weight:

**It floats in water**

**Principle of flotation**



Temperature > **Curie temperature**

Ferromagnetic → Paramagnetic

(Magnetic materials lose their ferromagnetic properties)

Temperature > **Néel temperature**

Antiferromagnetic → Paramagnetic

(Magnetic materials lose their antiferromagnetic properties)

Temperature < Debye temperature ( $T_D = \frac{h\nu_D}{k_B}$ , where  $\nu_D \rightarrow$  Debye frequency)



The temperature of a crystal's highest normal mode of vibration

The heat capacity of crystals increases linearly with **(Temperature)<sup>3</sup>**

We are the representatives of the cosmos; we are an example of what hydrogen atoms can do, given 15 billion years of cosmic evolution.

**Carl Sagan**

The **decay width** ( $\Gamma$ ) of a decaying particle with a **lifetime** ( $\tau$ ) is given by:  $\Gamma = \frac{h}{2\pi\tau}$

$$\Gamma \propto \frac{1}{\tau}$$

Electron + proton  $\rightarrow$  neutrino + neutron (**inverse beta decay**)



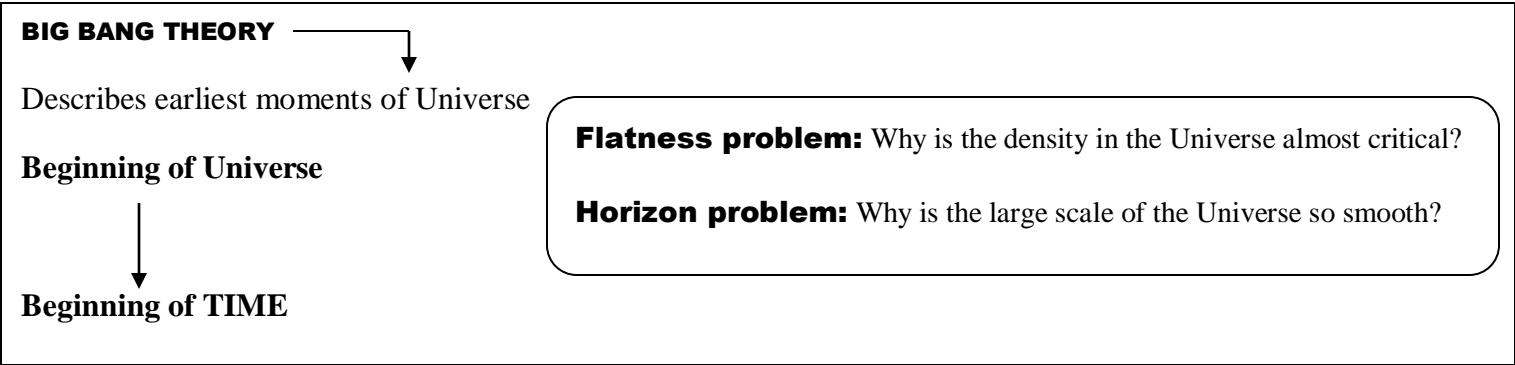
Takes place in stars of extremely high density

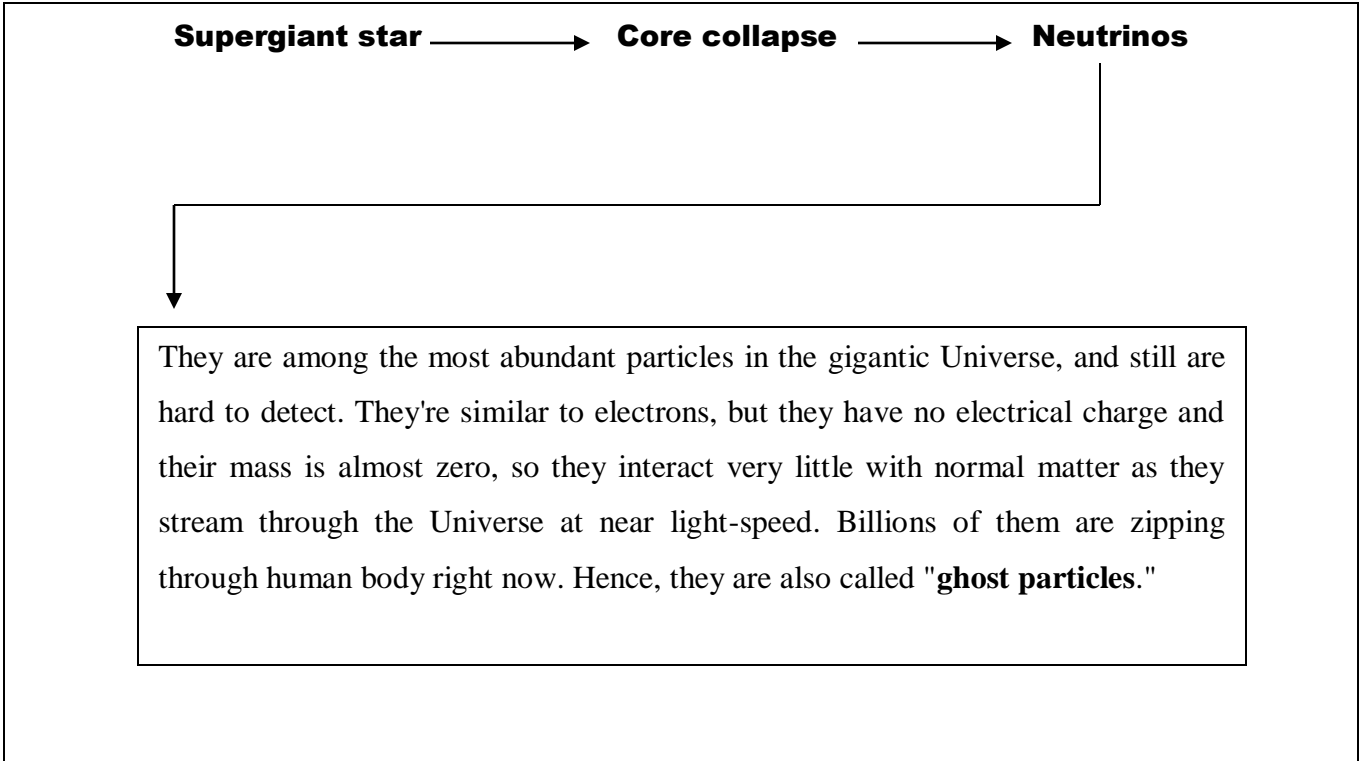
**Albert Einstein** Did Not Win the Nobel Prize for His Theory of Relativity in 1921 but For the Photoelectric Effect

**Albert Einstein's general theory of relativity** suggests that the sun's gravity bends the path of light from distant stars. It's a testable prediction, but only during a total solar eclipse.

- Albert Einstein's general theory of relativity was proved in 1919 during a solar eclipse.
- It took 15 years for Albert Einstein's Relativity theory to make him famous.

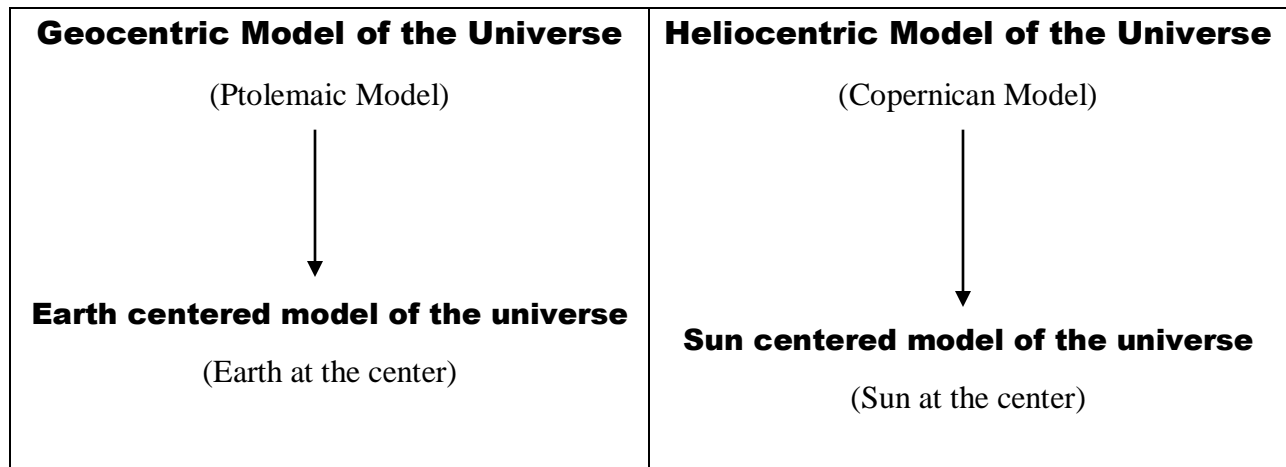
Einstein presented his general theory of relativity in 1916, but for an entire century nobody could find physical proof of black holes. In 2016, scientists finally detected gravitational waves that emitted from two black holes colliding, proving that such weird things not only exist, but that Einstein was right all along.





| <b>How does the universe work?</b>  | <b>Why does it work that way?</b>  |
|---|--|
| <ul style="list-style-type: none"> <li>• Observe the universe</li> <li>• Precise measurements</li> <li>• Is it testable?</li> </ul> | <ul style="list-style-type: none"> <li>• Look for abstract ideas</li> <li>• Mathematical models</li> <li>• Unified descriptions</li> </ul> |
| <p>⏟</p> <p><b>Experimental physics</b></p>   | <p>⏟</p> <p><b>Theoretical physics</b></p>   |

- In 1962, The U.S. Blew Up A **Hydrogen Bomb** In Space That Was 100 Times More Powerful Than Hiroshima.
- Astronauts on the **International Space Station** Witness around 15 Sunrises and 15 Sunsets Every Day.
- In 1977, We Received A Signal From **Deep Space** That Lasted 72 Seconds. We Still Don't Know How Or Where It Came From.



A cosmic mystery of immense proportions, once seemingly on the verge of solution, has deepened and left astronomers and astrophysicists more baffled than ever. The crux ... is that the vast majority of the mass of the universe seems to be missing.

**[Reporting a Nature article discrediting explanation of invisible mass being due to neutrinos]**

**— William J. Broad**

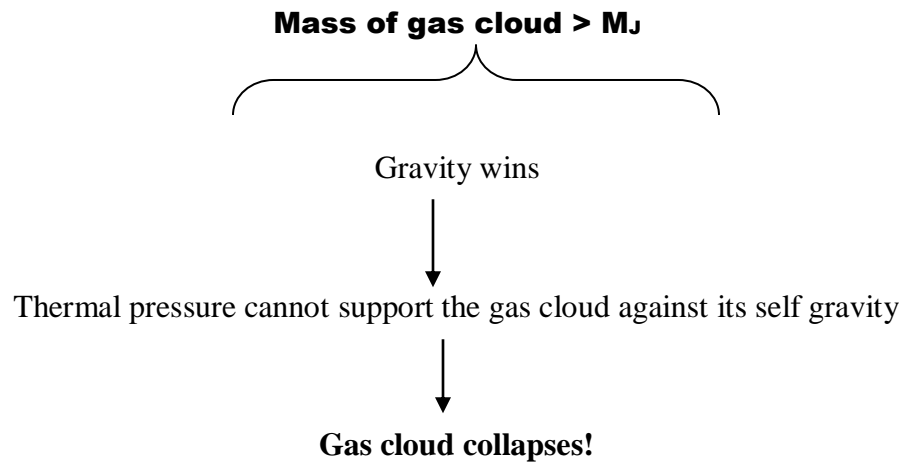
If you leave at Age of 15 in a Spaceship at Speed of Light and Spends **5 Years in Space**, when you get back on Earth you will 20 Years old. But all of your Friends who were 15 when you Left, will be 65 Years Old at that Time.

If you fall into black hole, you will able to see both the Universe beginning and ending due to **Time Dilation**.

**Jeans mass:**

$$M_J = \frac{3k_B TR}{2Gm}$$

$k_B$  = Boltzmann constant,  $T$  = temperature in Kelvin,  $R$  = radius of gas cloud,  $m$  = mass of gas particle and  $G$  = gravitational constant

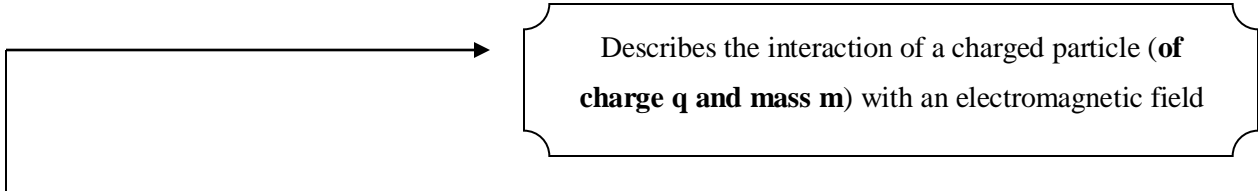


**Virial Theorem:**  $2K + U = 0$

- If  $2K > U$ : the gas pressure will dominate over gravity
- If  $2K < U$ : the gas cloud will collapse

**Low mass star** → cooler, fainter, long lifetime

**High mass star** → hotter, brighter, short lifetime



Describes the interaction of a charged particle (of charge  $q$  and mass  $m$ ) with an electromagnetic field

**Lorentz force law** for the force  $F$  acting on the particle:

$$F = q(E + v \times B)$$

where  $E$  and  $B$  are electric and magnetic field vectors and  $v$  is the velocity of the particle.

$$\% \text{ mass increase} = \frac{m - m_0}{m_0} \times 100 = (\text{Lorentz factor} - 1) \times 100$$

**Intensity:**

$$I = \epsilon_0 c^2 (E \times B)$$

Since:  $c = \frac{E}{B}$

$$I = \epsilon_0 c E^2$$

$I \propto E^2$

- The Sun's Mass Takes Up 99.86% Of The Solar System
- One Million Earths Can Fit Inside The Sun
- There Are More Trees On Earth Than Stars In The Milky Way
- The Sunset On Mars Appears Blue
- There Are More Stars In The Universe Than Grains of Sands on Earth
- One Day on Venus Is Longer Than One Year on Earth.
- Venus is the hottest planet in the solar system and has an average surface temperature of around 450° C.

$$\Delta A \times \Delta B \geq \frac{\hbar}{2}$$

where  $A$  and  $B$  are two conjugate variables in quantum mechanics

- position and momentum
- angular orientation and angular momentum
- energy and time

You can get an idea of human nature only when you can see the relationship of the individual human being to the whole cosmos.

**Rudolf Steiner**

Planck mass =  $1.2 \times 10^{19}$  GeV

about 22μ gram – much heavier than any mass of existing elementary particles

**Binary Stars** – a pair of stars in orbit around their common center of gravity.

- **Apparent Magnitude** – a star's brightness as it appears to Earth
- **Absolute Magnitude** – how bright a star actually is.

$$(\text{Apparent Magnitude} - \text{Absolute Magnitude}) = 5 \log_{10} \left( \frac{\text{Distance between the star and the earth}}{10} \right)$$

Distance modulus

Measured in parsecs ←

- **Red Giants** – a large, cool star of high luminosity
- **Super giants** – a very large, very bright red giant star
- **Nebulae** – clouds of dust and gases in space

- Hydrogen in core is depleted.
- Core contracts and heats up.
- Heat causes the outer layers to expand.
- Expanding causes the layers to cool.

A star's color reveals its surface temperature.

|     |        |         |
|-----|--------|---------|
| Sun | Yellow | 5,500 K |
|-----|--------|---------|

**Stars are classified by their spectra as:**

- O, B, A, F, G, K, and M spectral types

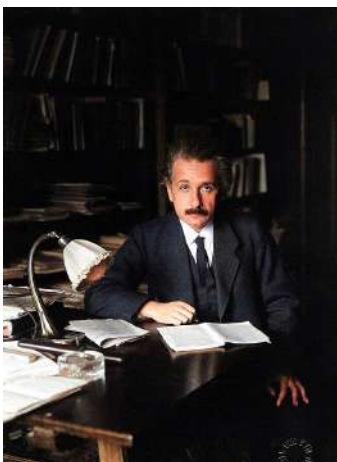
|                      |
|----------------------|
| <b>O B A F G K M</b> |
| hottest to coolest   |
| bluish to reddish    |

|          |                      |
|----------|----------------------|
| <b>O</b> | hotter than 25,000 K |
| <b>B</b> | 11,000 - 25,000 K    |
| <b>A</b> | 7500 - 11,000 K      |
| <b>F</b> | 6000 - 7500 K        |
| <b>G</b> | 5000 - 6000 K        |
| <b>K</b> | 3500 - 5000 K        |
| <b>M</b> | cooler than 3500 K   |

**Hertzsprung - Russell diagram** →

Graph of **luminosity** (or absolute magnitude) versus **temperature** (or spectral class)

|                      |               |
|----------------------|---------------|
| <b>How Far Away?</b> | Distance      |
| <b>How Bright?</b>   | Luminosity    |
| <b>How Hot?</b>      | Spectral Type |
| <b>How Massive?</b>  | Mass          |



Many of the stars in our universe come in pairs. Ordinary stars orbiting around a black hole will appear to "wobble" in the sky.



## Parallax Formula:

$$P = \frac{1}{d}$$

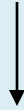
- P = the parallax angle of star (in arcseconds)
- d = the distance to star (in parsecs)

The closer the star, the more its apparent position shifts as observed from earth.

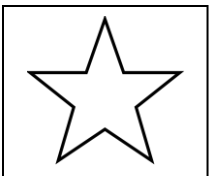
**1 parsec = 3.26 light years**

## Hooke's Law:

The force needed to stretch or compress a spring by some distance is proportional to that distance



To stretch or compress a spring twice as far, you need to apply twice the force.



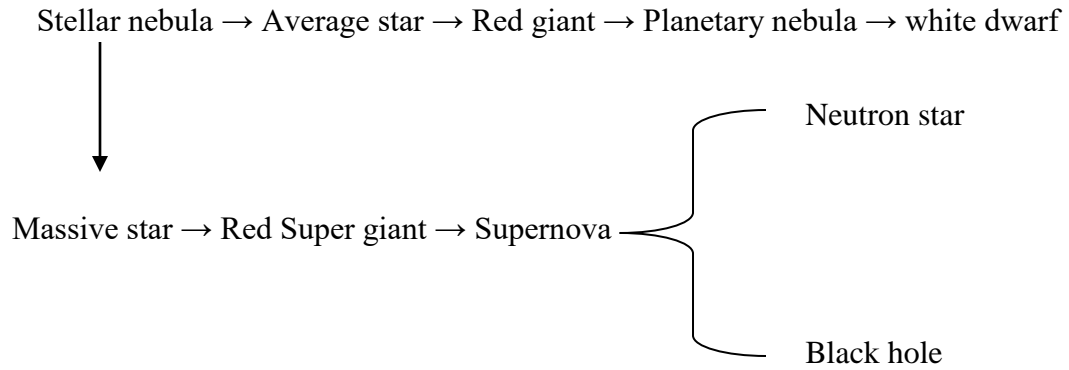
**Red color** → cooler → lower energy radiation → lower luminosity

**Blue color** → hotter → higher energy radiation → higher luminosity

For an object traveling at speed "v" in a circular path of radius "r", the magnitude of centripetal acceleration is:

$$a_{\text{centripetal}} = \frac{v^2}{r}$$

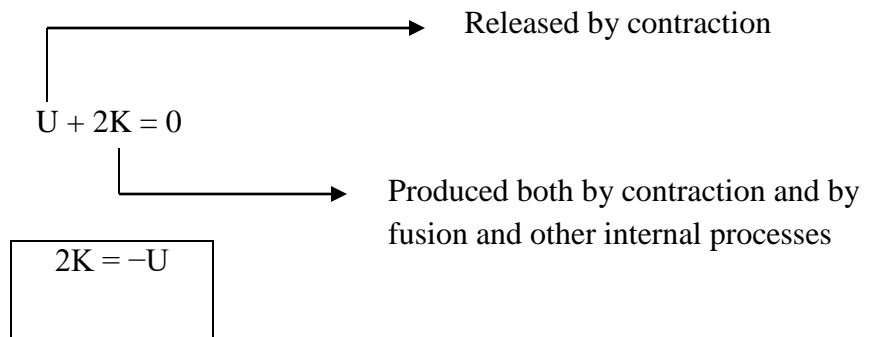
Centripetal acceleration is greater at high speeds and in smaller radius



### Life Cycle of a star

- A **quasar** is an enormously bright, distant galaxy with a giant black hole at its center.
- A **black dwarf** is a theoretical stellar remnant, specifically a white dwarf that has cooled sufficiently that it no longer emits significant heat or light.
- A **supernova** is a powerful and luminous stellar explosion.

### The virial theorem:



**The negative gravitational energy of a star is equal to twice its thermal energy.**

- A decrease in total energy  $E$  of the star leads to a decrease in  $U$  but an increase in  $K$  and hence temperature  $T$  increases, i.e. when a star loses its total energy, it heats up.

The rate of change of the total energy of star



(rate of nuclear energy generation in the deep interior – rate of energy loss in the form of radiation from the surface)

**In a state of thermal equilibrium:**

rate of nuclear energy generation in the deep interior = rate of energy loss in the form of radiation from the surface

**The total energy of star remains constant**

**4 possible sources of energy generation in stars:**

- cooling
- contraction
- chemical reactions
- nuclear reactions

If the temperature in the contracting core reaches values close to  $10^{10}$  Kelvin, the energy of the photons becomes large enough to break up the heavy nuclei into lighter ones – in particular  $^{56}\text{Fe}$  is disintegrated into  $\alpha$  particles and neutrons:



For ultrarelativistic particles ( $v \sim c$ ) the rest mass of the particle may be neglected and the equation:

$$E = \sqrt{p^2 c^2 + m_0^2 c^4}$$

take the form:

$$E \approx k_B T = pc$$

$$\lambda \approx \frac{hc}{k_B T}$$

**Gas pressure:**

$$P_{\text{gas}} \propto \rho T$$

**Radiation pressure:**

$$P_{\text{rad}} \propto T^4$$

**Degeneracy pressure** → resistance of electrons (or neutrons) against compression into a smaller volume.

$$P_{\text{deg}} \propto \rho^{5/3} \quad (\text{Non-relativistic case})$$

$$P_{\text{deg}} \propto \rho^{4/3} \quad (\text{Relativistic case})$$

|   |   |
|---|---|
| $M < 0.3M_{\text{sun}}$                     | Star completely convective  |
| $0.3M_{\text{sun}} < M < 1.5M_{\text{sun}}$ | <ul style="list-style-type: none"> <li>• Core radiative</li> <li>• Envelope convective</li> </ul> |
| $M > 1.5M_{\text{sun}}$                     | <ul style="list-style-type: none"> <li>• Core convective</li> <li>• Envelope radiative</li> </ul> |

- **Brown dwarfs** → heavier than a planet ( $13 \times$  mass of Jupiter) and lighter than a star.

**Hydrogen burning:**  $4p + 2e^- \rightarrow {}^4\text{He} + 2\nu_e$

- proceeds by pp chains and CNO cycle
- no heavier elements formed because no stable isotopes with mass number  $A = 8$
- neutrinos from proton → neutron conversion
- typical temperature  $10^7$  K ( $\sim 1$  keV)

**Helium burning:**  ${}^4\text{He} + {}^4\text{He} + {}^4\text{He} \leftrightarrow {}^8\text{Be} + {}^4\text{He} \rightarrow {}^{12}\text{C}$

- triple alpha reaction builds up Be with concentration  $\sim 10^9$



- typical temperature  $10^8$  K ( $\sim 10$  keV)

**Carbon burning:**

- Many reactions like  ${}^{12}\text{C} + {}^{12}\text{C} \rightarrow {}^{20}\text{Ne} + {}^4\text{He}$  etc.
- typical temperature  $10^9$  K ( $\sim 100$  keV)

A star generally cannot reach hydrostatic equilibrium if its surface is too cool.

## Kepler's First Law



The planet's orbit is an ellipse, so as the planet goes around its orbit, the distance between planet and the Sun is continually changing.

## The frame-dragging effect

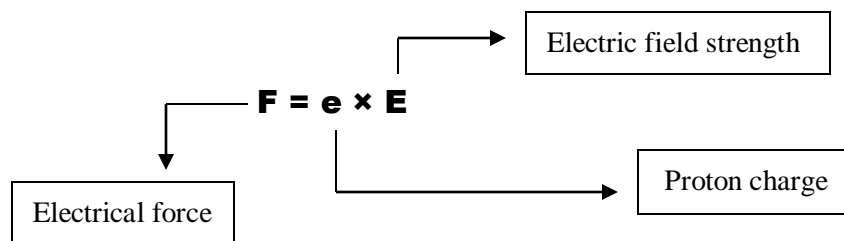


The extent to which a rotating object drags space and time along with it

## The geodetic effect



The distorting of space and time caused by a massive body's gravitational field (in this case, Earth)



Electrical force on charge "e"  $\propto$  Strength of electric field produced by charge "e"

- If the distance between nuclei  $< 10^{-15}\text{m}$ , the strong nuclear force overpowers the electromagnetic repulsion.

**Hydrostatic equilibrium keeps the fusion process at a constant rate:**

- If the nuclear fusion process speeds up:

More energy would be produced and pressure would increase. This increased pressure would cause the core to expand and cool, and the fusion rate would slow down to normal.

- If the core temperature drops:

The nuclear fusion process slows down

The pressure would decrease and the core would contract

As the core shrinks

Temperature would increase

Fusion rate would return to normal

The force of electrostatic repulsion between 2 protons is:

$$F = \frac{e^2}{4\pi\epsilon_0 r^2}$$

and the potential energy is:

$$E_p = \frac{e^2}{4\pi\epsilon_0 r}$$

So if  $\frac{3}{2} k_B T = \frac{e^2}{4\pi\epsilon_0 r}$ , the two protons can exceed the electrostatic force of repulsion and fuse together.

$$r_{\min} = r = \frac{e^2}{6\pi\epsilon_0 k_B T}$$

$r_{\min}$  → the distance of closest approach at which the nuclear attractive force becomes dominant to bind the two protons together.

- At low velocities, the electrostatic force of repulsion prevents the collision of protons.
- At high velocities, protons come close enough for the strong nuclear attractive force to bind them together.

There is a small (but non-zero) probability that two protons can overcome their repulsion and fuse even if their velocities are too low. This is called **quantum mechanical tunneling**.

$$\text{Probability} \propto \exp\left(-\frac{r_{\min}}{\lambda}\right)$$

$$\frac{mv^2}{2} = \frac{3}{2} k_B T$$

$$\lambda = \frac{h}{\sqrt{3mk_B T}}$$

$$\text{Probability} \propto \exp\left(-\frac{r_{\min}}{\lambda}\right)$$



$$\text{Probability} \propto \exp\left(-\frac{\text{Fine structure constant}}{\pi} \times \sqrt{\frac{mc^2}{3k_B T}}\right)$$

**Probability** is highest at fast speeds  $v$  (or high temperatures  $T$ )

- $\nu = \frac{k_B T_{\text{core}}}{h}$  is the frequency corresponding to the core temperature of the star.
- For a photon traveling from the center of Sun to the surface, with constant mean free path " $\ell$ " and assuming no destruction and recreation, the time taken for it to traverse this path is:  $t = \frac{R_{\text{Sun}}^2}{c\ell}$ , where  $c$  is the speed of light and  $R_{\text{Sun}}$  is the radius of the sun.

For  $\ell = 3.01 \times 10^{-5}$  m:

$$t = 5.36 \times 10^{13} \text{ s} = 1.70 \times 10^6 \text{ yr.}$$

If the photon didn't interact at all with matter, then  $t = \frac{R_{\text{Sun}}}{c}$ .

### Hawking Radiation:

- Proposed by Professor Stephen Hawking in 1974.
- Reduces mass and energy of a black hole.

Our Moon is constantly moving away from Earth whereas the Andromeda Galaxy is constantly moving towards Earth. Millions of years from now, people will see a tiny Moon but a huge galaxy in the night sky.

## Thermodynamics

1. Energy is conserved
2. Randomness increases
3. Absolute Zero temperature is Unattainable

Because:

$$T_{\text{BH}} \propto \frac{1}{M}$$

- Tiny Black Hole is hot
- Big Black Hole is cold

In 2012, the Quantum Physicists **Dr. S. Haroche** and **Dr. D. Wineland** received Nobel Prize for their experiments which showed that "A Particle can be at two different locations at the same time." These Experiments indirectly showed that "**Parallel Universe do Exist!**"

Neutron stars are so dense that one teaspoon of their contents weighs more than **Mount Everest**.

As mass decreases, temperature increases, and the radiation becomes more intense.

60% Dark Energy (we don't know what it is)  
35% Cold dark matter (we don't know what it is)  
5% Nuclei and electrons (visible as stars ~0.5%)

**Venus** looked just like earth 2 billion years ago

Objects that travel with  $v \ll c$  obey this relation:

$$z = \frac{\Delta\lambda}{\lambda} = \frac{v}{c}$$

- Objects moving away from observer  $\rightarrow$  frequency decreases  $\rightarrow$  wavelength increases (**red shift**)
- Objects moving towards observer  $\rightarrow$  frequency increases  $\rightarrow$  wavelength decreases (**blue shift**)

**Einstein Theory**  $\rightarrow$  4 dimensions (length, width, depth, and time)

**String theory**  $\rightarrow$  4 dimensions + 7 other dimensions

11<sup>th</sup> dimension holds the universe together

**The black hole no hair theorem:**

Mass, charge, and angular momentum are the only properties a black hole can possess

The Sky is Dark at Night  $\rightarrow$  there must be some limit to the observable Universe.

Density parameter ( $\Omega$ ) =  $\frac{\text{density}}{\text{critical density}}$

- $\Omega > 1 \rightarrow$  closed universe
- $\Omega = 1 \rightarrow$  flat universe
- $\Omega < 1 \rightarrow$  open universe

**Gamma radiation**  $\rightarrow \lambda < 0.001 \text{ nm}, E_\gamma > 1.24 \text{ MeV}$



Produced in nuclear reactions and other very high energy processes

**X-rays**  $\rightarrow 0.001 \text{ nm} < \lambda < 10 \text{ nm}, 124 \text{ eV} < E_{\text{x-ray}} < 1.24 \text{ MeV}$



Produced in supernovae remnants and the solar corona, as well as in the hot gas between galaxy clusters

**Thomson Scattering** ( $h\nu \ll m_0c^2$ )

The photon and electron just both bounce off each other, changing their direction, but there is no exchange of energy

**Compton scattering** ( $h\nu > m_0c^2$ )

A photon of high energy collides with a stationary electron and transfers part of its energy and momentum to the electron, decreasing its frequency in the process

The maximum energy gained by photons via **inverse Compton scattering** is proportional to its initial energy multiplied by the square of twice the Lorentz factor (where the **Lorentz factor** is

given by  $\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$  and  $v$  is the velocity of the electron):

$$E_{\max} = (h\nu)_{\max} \propto 4 \gamma^2 h\nu_0$$

- $1 \text{ eV} = 1.602 \times 10^{-19} \text{ J} = 1.602 \times 10^{-12} \text{ erg}$

**Brown dwarf**

- Too big to be a planet
- Too small to be a star

**Pulsars** → Rotating neutron stars emitting beams of particles and electromagnetic radiation

- **Gravity** warps space and time
- Even photons with no mass can have their trajectories bent → **gravitational lensing**

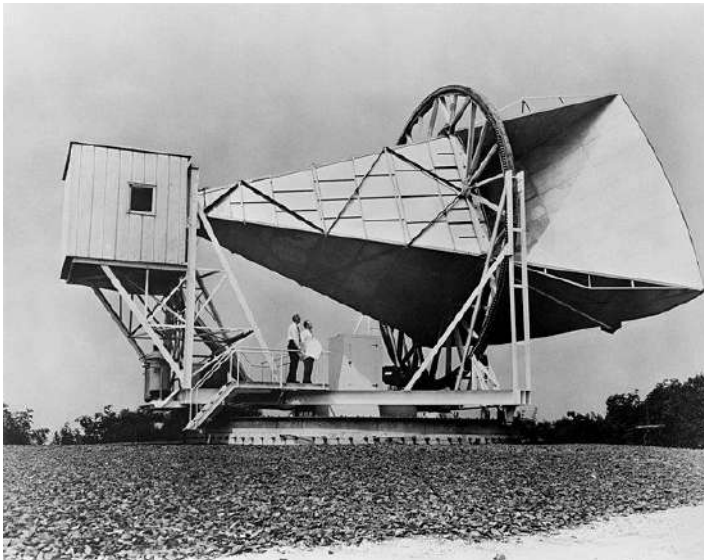
|                     |                                      |                               |
|---------------------|--------------------------------------|-------------------------------|
| $\lambda \ll R$     | $\sigma \approx \pi R^2$             | <b>Geometrical scattering</b> |
| $\lambda \approx R$ | $\sigma \propto \frac{1}{\lambda}$   | <b>Mie scattering</b>         |
| $\lambda \gg R$     | $\sigma \propto \frac{1}{\lambda^4}$ | <b>Rayleigh scattering</b>    |

where  $R$  is the particle's radius,  $\lambda$  is the wavelength of the light and  $\sigma$  is the scattering cross section

$$\text{Energy of radiation} \propto \frac{1}{(\text{scale factor of the universe})}$$

$$\text{Energy density of radiation} \propto \frac{1}{(\text{scale factor of the universe})^4}$$

$$\text{Relative brightness} = \frac{\text{absolute brightness}}{(\text{distance})^2}$$



In 1964, **Arno Penzias and Robert Wilson**, two engineers at Bell Labs in New Jersey discovered Cosmic Background radiation when trying to get rid of noise from an antenna aimed at telecommunications satellites.

$$\frac{d(\text{photons})}{dt} = \text{Metric} + \text{Compton Scattering}$$

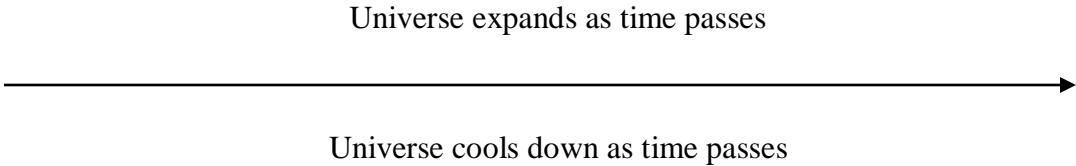
$$\frac{d(\text{electrons+hadrons})}{dt} = \text{Metric} + \text{Compton Scattering} + \text{Weak Interaction}$$

$$\frac{d(\text{neutrinos})}{dt} = \text{Metric} + \text{Weak Interaction}$$

Spacetime tells matter how to move, matter  
tells spacetime how to curve

**Special Relativity:**

- The speed of light is the same for any observer



- If A couples to B, and B to C, A should couple to C.

A force's impact on an object is influenced by both how long it acts and how intense it is.

At scale  $L \sim \sqrt{\frac{G\hbar}{c^3}}$ , energy fluctuations become so large that even spacetime geometry is no longer smooth at all.

**3 types of geometries for our universe:**

- Hyperbolic (negative curvature)
- Elliptic (positive curvature)
- Euclidean (zero curvature)

Photon + Hydrogen atom  $\rightarrow$  proton + electron

**(photodissociation)**

Proton + electron  $\rightarrow$  Photon + Hydrogen atom

**(radiative recombination)**

**General theory of relativity** { Equivalence between inertial mass and gravitational mass

|                      |   |
|----------------------|---|
| <b>Big Bang</b>      | Birth of the Universe                                 |
| <b>Planck Era</b>    | String Theory / Quantum Cosmology                     |
| <b>Inflation Era</b> | Symmetry Breaking $\rightarrow$ Exponential Expansion |
| <b>Quark Era</b>     | Free Quarks in Thermal Equilibrium                    |
| <b>Hadron Era</b>    | Matter Anti Matter Asymmetry                          |
| <b>Lepton Era</b>    | Rapid Expansion/cooling (leptons/photons equilibrium) |



|                         |                                     |
|-------------------------|-------------------------------------|
| <b>Radiation Era</b>    | Nucleosynthesis, Decoupling         |
| <b>Matter Era</b>       | Structure Formation, first galaxies |
| <b>Acceleration Era</b> | Acceleration phase of the Universe  |

**Newton Theory:**

Weight is proportional to Mass

**Einstein Theory:**

Energy is proportional to Mass

**Neither explained origin of Mass**

- **Electroweak theory** predicted a heavy version of the photon called the  $Z^0$  which was discovered in 1983.
- **Quantum field theory** which postulates that matter is composed out of elementary particles bound together by forces, mediated by exchange of other elementary particles.

An alien star passed through our solar system 70,000 years ago. Astronomers have discovered that a red dwarf came 0.8 light-years from the sun around the time modern humans are thought to have first spread across Asia. This is the "**closest known flyby of a star**" in our solar system.

|         |    |
|---------|----|
| Mercury | 0  |
| Venus   | 0  |
| Earth   | 1  |
| Mars    | 2  |
| Jupiter | 66 |
| Saturn  | 62 |
| Uranus  | 27 |
| Neptune | 13 |

**There are more than  
171 moons in our  
solar system**

If the earth was the size of a marble you would need seven miles of space to build a scale model of the solar system.

$$S = k_B \ln W$$

**For Black hole:**

$$S = \frac{k_B A}{4L_{\text{Planck}}^2}$$

$$\frac{A}{4L_{\text{Planck}}^2} = \ln W$$

If  $A = L_{\text{Planck}}^2$ :

$$W = 1.28402541669$$

**For The Smallest Known Black Hole**

Black hole entropy is the amount of entropy that must be assigned to a black hole in order for it to comply with the laws of thermodynamics as they are interpreted by observers external to that black hole.

— **Jacob Bekenstein**

- 33 light-years away there is an **exoplanet** completely covered burning ice.
- You can't cry on space because your tears won't ever fall.
- There is a **water reservoir** floating in space that is equivalent to 140 trillion times all the water in the world's ocean.

$$\text{Electron relativistic mass} = \frac{\text{electron rest mass}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

If Electron relativistic mass = proton rest mass:

$$\sqrt{1 - \frac{v^2}{c^2}} = \frac{\text{electron rest mass}}{\text{proton rest mass}} = \frac{1}{1836.1567343}$$

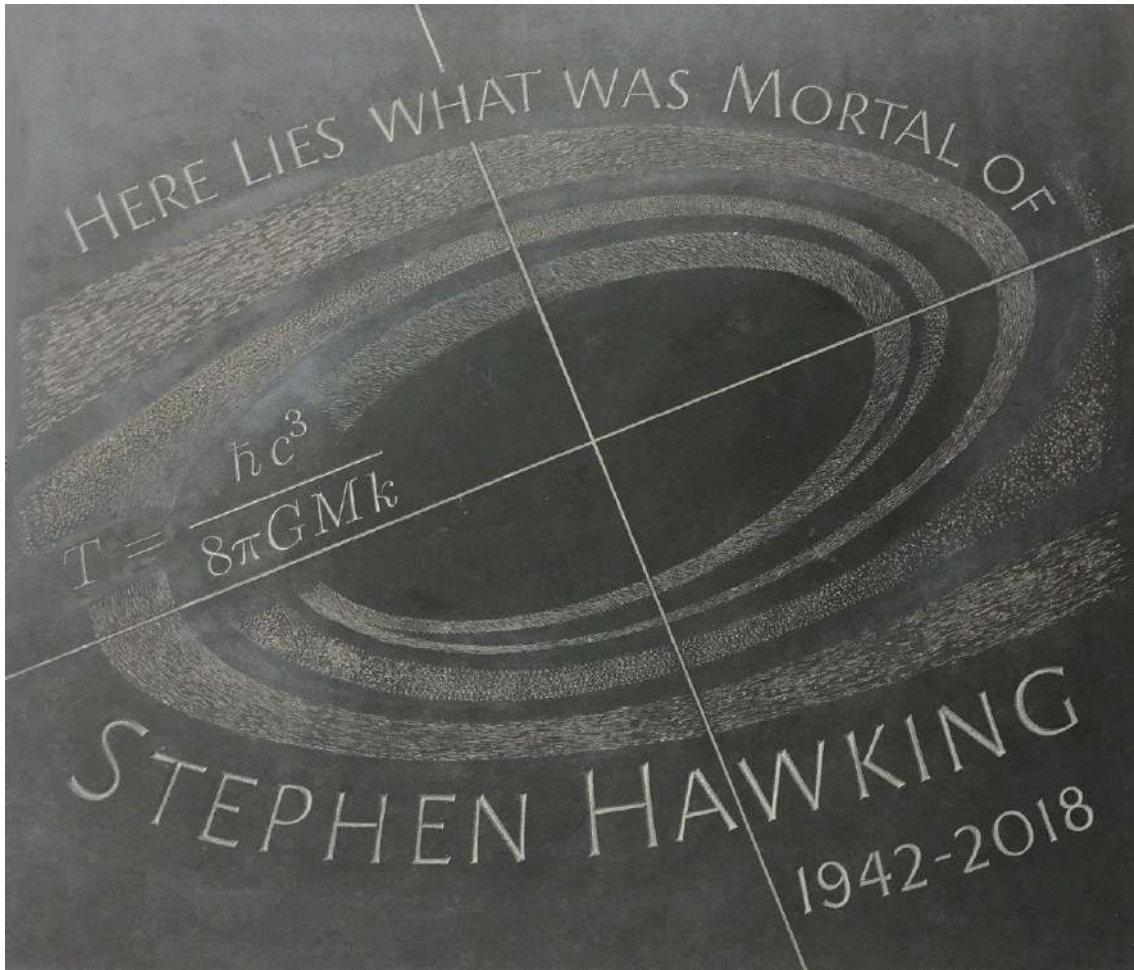
$$v = 0.99999985169c$$

**A electron must travel at a velocity  $v = 0.99999985169c$  so that its relativistic mass is equal to the rest mass of the proton**

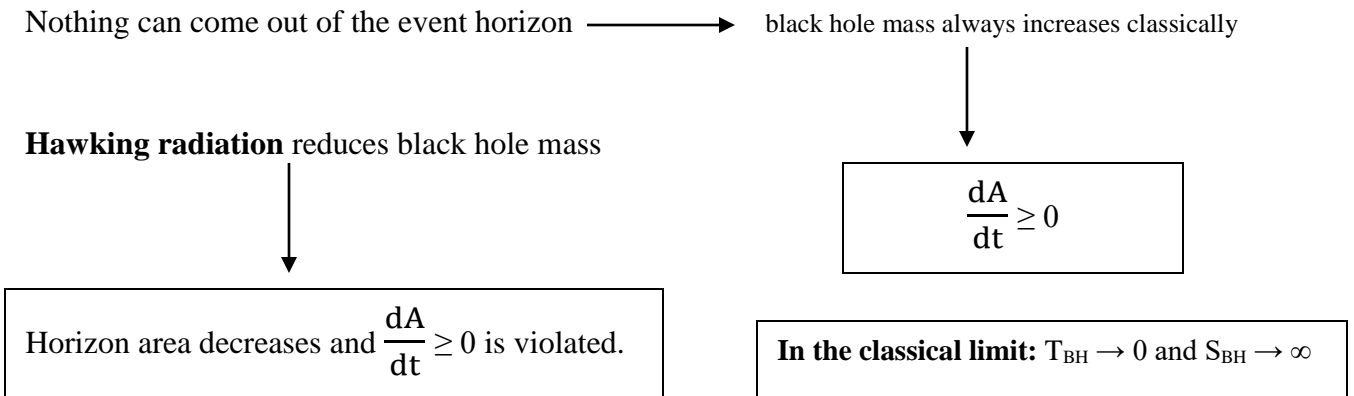
We don't have any real pictures of the Milky Way galaxy. Most non-illustrated images of the entire Milky Way spiral are actually of another spiral galaxy called **Messier 74**. It's impossible to take a full photo of the Milky Way's spiral structure because it's about 100,000 light-years across, and we're stuck on the inside.

Pluto was named by an 11-year-old British girl. One day, Venetia Burney was having breakfast with her grandpa, who was reading about the new planet and wondering what it would be called. She had read about Greek and Roman legends, so she asked him, "**Why not call it Pluto?**" Grandpa later passed on her suggestion to an astronomer at Oxford University, and the rest is history.

Einstein was once asked how it felt to be the smartest man alive. Einstein's reply was "**I don't know, you'll have to ask Nikola Tesla.**"



A tombstone of the crypt with Hawking's ashes with the formula ( $T_{\text{BH}} = \frac{\hbar c^3}{8\pi G M k_{\text{B}}}$ ) for the black hole temperature, placed in the flooring at Westminster Abbey in London.



**No hair theorem:**

Black holes are characterized by 3 quantities:

**mass, charge, and angular momentum**

For black hole with 10 solar mass:

Temperature  $\sim 6 \times 10^9$  K (**very low temperature**)

Entropy  $\sim 10^{79} k_B$  (huge entropy)

Entropy of sun  $\sim 10^{58}$

**Spin parameter "a"** cannot exceed **black hole mass "M"**: for  $a > M$  the Kerr metric describes a naked singularity.

**Hawking 1975:** Black hole background + Quantum Field theory  $\rightarrow$  Black hole emits radiation!!

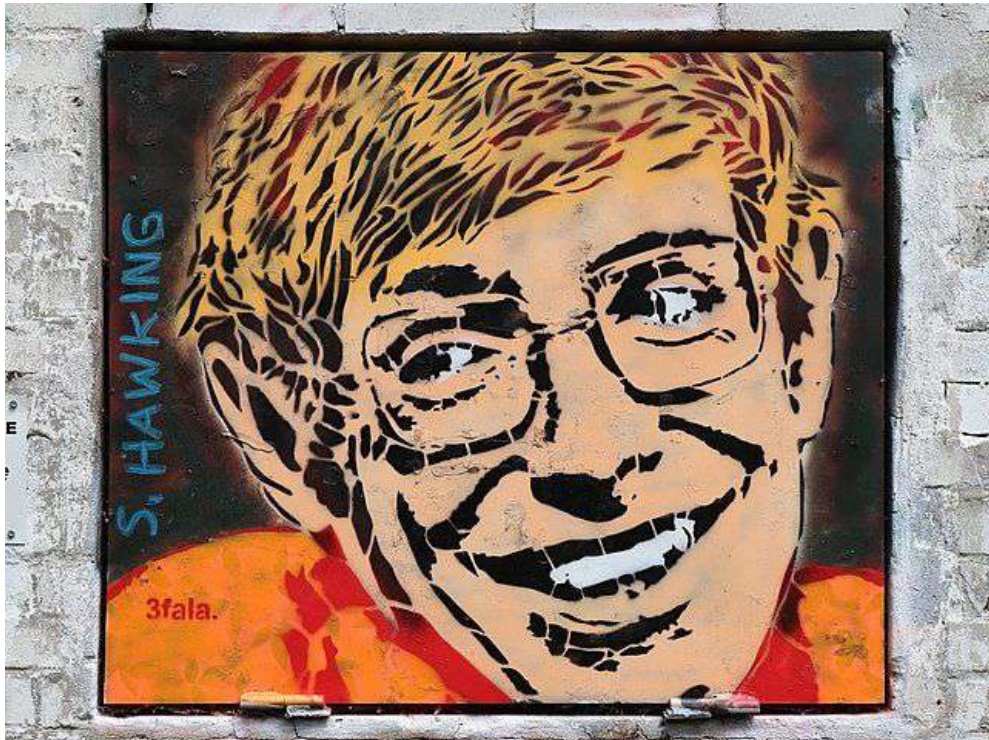
**Hawking 1976:** Black hole as a quantum pure state + hawking radiation  $\rightarrow$  Unitarity of Quantum Mechanics is broken!!

- **Solar Mass Black Hole:**  $T_{BH} \sim 10^{-7}$  K,  $t_{evap} \sim 10^{66}$  yr
- **Black Hole from Large Hadron Collider:**  $T_{BH} \sim 10^{44}$  K,  $t_{evap} \sim 10^{-80}$  s

|                            |  |                                   |
|----------------------------|--|-----------------------------------|
| Information is indeed lost | Information is stored in Hawking radiation | Evaporation stops at Planck scale |
| Quantum Mechanics is wrong | General Relativity is wrong                | Statistical Mechanics is wrong    |

Quantum Field Theory is valid up to  $10^{-16}$ cm

- Particles collide in three dimensional space.
- As the particles approach in a particle accelerator, their gravitational attraction increases steadily.
- When the particles are extremely close, they may enter space with more dimensions.
- The extra dimensions would allow gravity to increase more rapidly so a black hole can form.
- Such a black hole would immediately evaporate, sending out a unique pattern of thermal radiation.



"This was a golden age, in which we solved most of the major problems in black hole theory even before there was any observational evidence for black holes. In fact, we were so successful with the classical general theory of relativity that I was at a bit of a loose end in 1973 after the publication with George Ellis of our book *The Large Scale Structure of Space–Time*. My work with Penrose had shown that general relativity broke down at singularities, so the obvious next step would be to combine general relativity—the theory of the very large—with quantum theory—the theory of the very small. In particular, I wondered, can one have atoms in which the nucleus is a tiny primordial black hole, formed in the early universe? My investigations revealed a deep and previously unsuspected relationship between gravity and thermodynamics, the science of heat, and resolved a paradox that had been argued over for thirty years without much progress: how could the radiation left over from a shrinking black hole carry all of the information about what made the black hole? I discovered that information is not lost, but it is not returned in a useful way—like burning an encyclopedia but retaining the smoke and ashes.

To answer this, I studied how quantum fields or particles would scatter off a black hole. I was expecting that part of an incident wave would be absorbed, and the remainder scattered. But to my great surprise I found there seemed to be emission from the black hole itself. At first, I thought this must be a mistake in my calculation. But what persuaded me that it was real was that the emission was exactly what was required to identify the area of the horizon with the entropy of a black hole. This entropy, a measure of the disorder of a system, is summed up in this simple formula which expresses the entropy in terms of the area of the horizon, and the three fundamental constants of nature,  $c$ , the speed of light,  $G$ , Newton's constant of gravitation, and  $\hbar$ , Planck's constant. The emission of this thermal radiation from the black hole is now called Hawking radiation and I'm proud to have discovered it."

— **Stephen Hawking**

- Neutron Star has a hard surface; the curvature is large - but finite.
- Black Hole: No Surface – curvature is infinite at the centre

Black holes are the harmonic oscillator of quantum gravity.

(A. Strominger)

### Stephen Hawking-Jacob Bekenstein debate (1972)

**Stephen Hawking:** the analogy between the area theorem and the second law of thermodynamics is just a matter of coincidence.

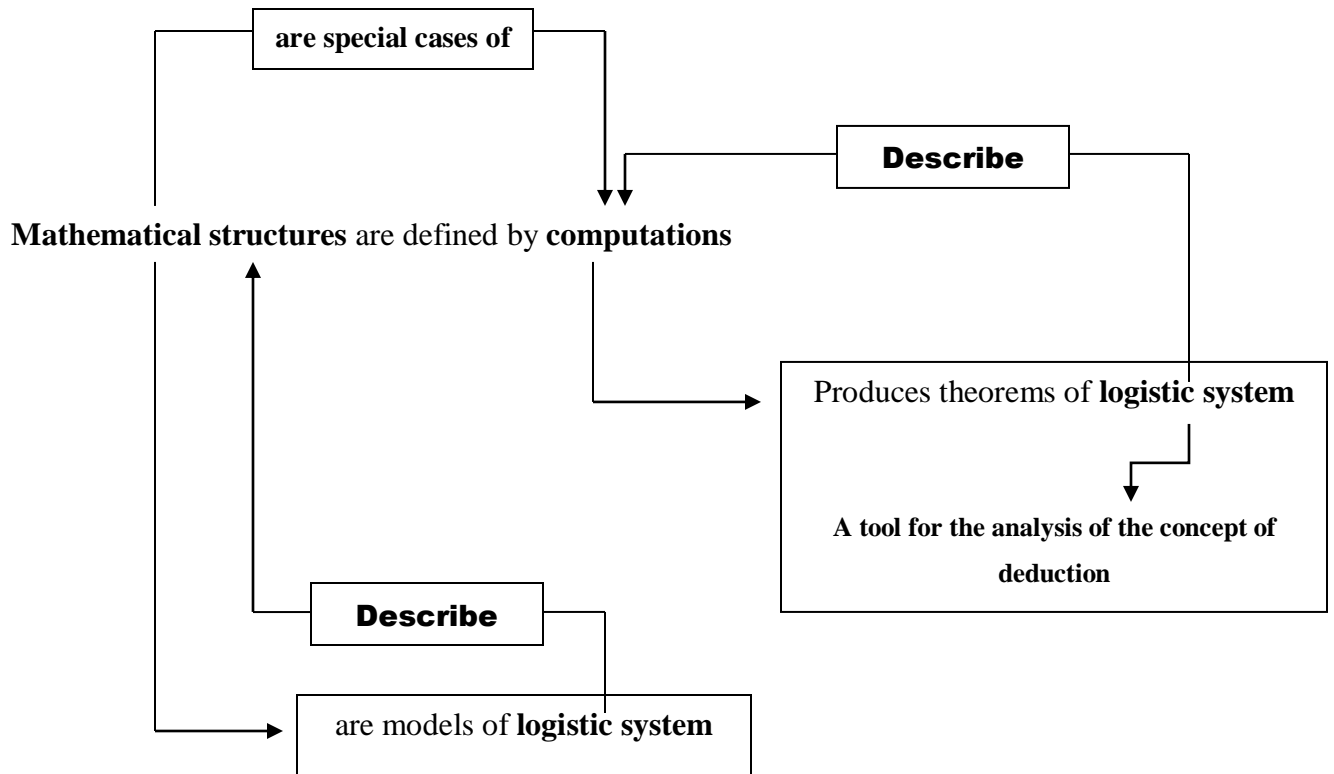
**Jacob Bekenstein:** I am not convinced. Nowhere in nature the second law of thermodynamics is violated. Why black holes would be an exception? I believe that the area of black holes is actually a manifestation of their entropy.

**John Wheeler** (Jacob Bekenstein's doctoral advisor) **to Bekenstein:** Your idea is just so crazy that it might actually be true.

**Stephen Hawking:** If a black hole has entropy, it must have a temperature, and if it has a temperature it must radiate like a blackbody. But if nothing can escape from a black hole, how can it radiate?

**1974:** end of the debate and discovery of the Hawking radiation. Professor Stephen Hawking admits that **Jacob Bekenstein** was right: Black Holes have entropy, which is proportional to their area.





**Theories of Origin of Life:**

|          |   |
|----------|---|
| <b>1</b> | Life formation on the earth may have been taken place due to supernatural entity  |
| <b>2</b> | Life formation did not take place on earth. It took place somewhere else in the space or on any other planet and carried to the earth               |
| <b>3</b> | Life formation on the earth could have arisen through a series of organic chemical reactions that produced ever more complex biochemical structures |
| <b>4</b> | Life may have evolved from non-living matter as association with prebiotic molecules under primitive earth conditions                               |

I learned very early the difference between knowing the name of something and knowing something.

— Richard P. Feynman

$$F = \frac{e^2}{4\pi\epsilon_0 r^2}$$

If  $r = \sqrt{\frac{\hbar G}{c^3}}$  :

$$F = \frac{e^2}{(\text{Planck charge})^2} \times \text{Planck force}$$

$$F = \left[ \frac{1.602176634 \times 10^{-19}}{1.87554603778 \times 10^{-18}} \right]^2 \times \text{Planck force}$$

$$F = \left[ \frac{1}{11.7062376144} \right]^2 \times \text{Planck force}$$

$$F = 0.00729735256 \times \text{Planck force}$$

So as the distance between the 2 electrons equals Planck length, the electric force between two electrons is 0.00729735256 times the Planck force.

If  $F = \frac{c^4}{G}$  :

$$\frac{c^4}{G} = \frac{e^2}{4\pi\epsilon_0 r^2}$$

$$r = \frac{e}{\text{Planck charge}} \times \text{Planck length}$$

$$r = 0.08542454313 \times \text{Planck length}$$

So as the electric force between two electrons equals the Planck force, the distance between the 2 electrons is 0.08542454313 times the Planck length.

On average, gravity on a neutron star is 2 billion times stronger than gravity on Earth. In fact, it's strong enough to significantly bend radiation from the star in a process known as **gravitational lensing**, allowing astronomers to see some of the back side of the star.

**Muons are subatomic particles that exist for just two millionths of a second. Scientists think they could be the key to many mysteries of the Universe.**

$$F = \frac{G \times \text{electron rest mass} \times \text{electron rest mass}}{r^2}$$

If  $r = \sqrt{\frac{\hbar G}{c^3}}$  :

$$F = \frac{\hbar c}{\lambda_C^2} \quad (\text{where } \lambda_C \text{ is the reduced Compton wavelength of the electron})$$

So as the distance between the 2 electrons equals Planck length, the gravitational force between

$$\text{two electrons} \propto \frac{1}{(\text{reduced compton wavelength of the electron})^2}$$

If  $F = \frac{c^4}{G}$ :

$$\frac{c^4}{G} = \frac{G \times \text{electron rest mass} \times \text{electron rest mass}}{r^2}$$

$$r = \frac{r_s}{2}$$

So as the gravitational force between two electrons equals the Planck force, the distance between the 2 electrons is half the Schwarzschild radius of the electron.

Solar radius =  $6.957 \times 10^5$  km

Schwarzschild radius of the sun =  $\frac{2GM}{c^2} = 3.0 \times 10^3$  m = 3 km

**Solar radius =  $(2.319 \times 10^5) \times$  Schwarzschild radius of the sun**

**Some scientists say that if space is truly infinite, then there is probably an exact copy of you somewhere in the universe.**

- $pc \ll m_0c^2 \rightarrow$  Classical particle
- $pc \gg m_0c^2 \rightarrow$  Ultrarelativistic particle
- $3 + 4 = 4 + 3$  (**commutative**)
- $3 - 5 \neq 5 - 3$  (**noncommutative**)
- $EA + T = EAT \neq TEA = T + EA$

**Richard Feynman used to spend up to 5 hours practicing his lectures in empty classroom**

**Frame dragging** is the idea that spacetime is elastic and particles in it will exchange energy. That means spacetime will absorb some of the energy of a spinning particle. Research studies have shown that Earth is dragging spacetime around it as it rotates.

| <b>Gravitoelectromagnetism</b>   | <b>Electromagnetism</b>          |
|--|----------------------------------|
| $F_g = m (E_g + v \times 4B_g)$  | $F_e = q (E + v \times B)$       |
| $S_g = -\frac{c^2}{8\pi G} E_g \times 4B_g$  | $S_e = c^2\epsilon_0 E \times B$ |
| <ul style="list-style-type: none"> <li>• <math>F_g</math> = the net (<b>Lorentz</b>) force acting on particle of mass <math>m</math> due to a gravitoelectromagnetic field</li> <li>• <math>v</math> = velocity of the particle and <math>c</math> = speed of light in vacuum</li> <li>• <math>E_g</math> = gravitoelectric field and <math>B_g</math> = gravitomagnetic field</li> <li>• <math>G</math> = gravitational constant and <math>S_g</math> = gravitoelectromagnetic Poynting vector</li> </ul> |                                  |
| <ul style="list-style-type: none"> <li>• <math>F_g</math> = the net (<b>Lorentz</b>) force acting on particle of charge <math>q</math> due to a electromagnetic field</li> <li>• <math>v</math> = velocity of the particle and <math>c</math> = speed of light in vacuum</li> <li>• <math>E</math> = electric field and <math>B</math> = magnetic field</li> <li>• <math>\epsilon_0</math> = absolute permittivity of free space and <math>S_e</math> = electromagnetic Poynting vector</li> </ul>         |                                  |

### Non-contact force

{ A force which acts on an object without coming physically in contact with it }

**All four known fundamental interactions are non-contact forces**

**Supercooling** is the process of lowering the temperature of a liquid or a gas below its freezing point without it becoming a solid.

### **Zero-energy universe hypothesis**

**The total amount of energy in the universe is exactly zero:** its amount of positive energy in the form of matter is exactly canceled out by its negative energy in the form of gravity

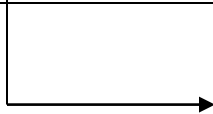
|                         |   |
|-------------------------|---|
| <b>Lambda-CDM model</b> | Big-Bang cosmological model with a <b>cosmological constant</b> and <b>cold dark matter</b> |
|-------------------------|---|

Provides an explanation for the universe's observed accelerating expansion

Provides an explanation for the dynamics of galaxies and galaxy

Due to the highly **elliptical orbit of Pluto**, it sometimes gets closer to the Sun than Neptune. In fact during the years 1979 to 1999, Neptune was the 9th Planet and Pluto was the 8th Planet from the Sun!

|                             |   |
|-----------------------------|---|
| <b>Eternal inflation</b>    | New universes pop into existence at an unknown rate – creating a complex web of bubble universes within a vast multiverse   |
| <b>Loop quantum gravity</b> | The universe is a network of intersecting quantum threads – each of which carries quantum information about the size and shape of nearby space  |
| <b>Graviphoton</b>          | A hypothetical particle which emerges as an excitation of the gravitational field in spacetime dimensions higher than four – as described in <b>Kaluza–Klein theory</b> .<br><br>↓<br><b>classical unified field theory of gravitation and electromagnetism</b> |



whose physical properties are virtually indistinguishable from a photon

**Ekpyrotic model of the universe**

Our current universe arose from the collision of two three-dimensional universes traveling in a hidden fourth spatial dimension. This model does not require a singularity at the moment of the **Big Bang**.

**Hartle-Hawking model**



Universe has no initial boundaries in time or space

**Fermions** (= matter): quarks and leptons  
**Bosons** (= interactions): gauge fields + Higgs boson (God's particle)  
 Venus and Uranus are the only planets that rotate **clockwise**. The other six planets in the solar system rotate **counterclockwise**.

**Naked singularity** → a hypothetical gravitational singularity without an event horizon

| <b>Weak Anthropic Principle</b>                      | <b>Strong Anthropic Principle</b>                       |
|--|---|
| If the world were any different we would not be here | The world had to be as it is in order for us to be here |
| ↓  | ↓   |
| The emergence of life is possible                    | The emergence of life is inevitable                     |

|                                 |  |
|---------------------------------|--|
| <b>absurd universe</b>          | Our universe just happens to be the way it is.   |
| <b>unique universe</b>          | There is a deep underlying unity in the laws of physics that make it necessary for the Universe being the way it is.   |
| <b>multiverse</b>               | The idea of multiple universes. Each of which comprise everything that exists: the entirety of space, time, matter, energy, information, and the physical laws and constants that describe them. |
| <b>Intelligent design</b>       | Life on earth is so complex that it cannot be explained by the scientific theory of evolution and therefore must have been designed by a supernatural entity.                                    |
| <b>self-explaining universe</b> | No phenomenon can be said to exist until it is observed.   |
| <b>fake universe</b>            | We are living in a simulated universe.   |

N → number of spatial dimensions

T → number of time dimensions

- If  $N > 3$  and  $T = 1$ : the orbit of a planet about its Sun cannot remain stable.



- If  $T > 1$ : the high energetic protons and electrons would be unstable and could decay into particles having greater mass than themselves.
  - A proton can decay into a neutron, a positron and a neutrino
  - An electron can decay into a neutron, an antiproton and a neutrino
- If  $N = 1$  and  $T = 3$ : all particles are **tachyons** with imaginary rest mass.

Only a  $(N + T) = (3 + 1)$  **dimensional universe** can contain dynamic observers who are complex and stable enough to be able to understand and predict all of space and time and their contents (including planets, stars, galaxies and all other forms of matter and energy) to any extent at all

**$T > 1$  or  $T < 1$ : insufficient predictability**

**$N > 3$ : insufficient stability**

**$N < 3$ : insufficient complexity**

|                               |   |
|-------------------------------|---|
| <b>1 dimensional universe</b> | made up of only 1 dimension (width)   |
| <b>2 dimensional universe</b> | made up of 2 dimensions (width and breadth)   |
| <b>3 dimensional universe</b> | made up of 3 dimensions (width, breadth and height)   |
| <b>4 dimensional universe</b> | made up of 4 dimensions (width, breadth, height and time)                                     |
| <b>5 dimensional universe</b> | more challenging to visualize because we ourselves cannot perceive dimensions $> 4$ around us |

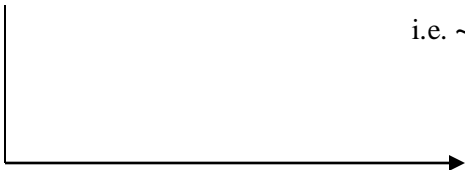
**Causality Principle:** all real events necessarily have a cause

**Dark matter** could warm certain planets in the place of a sun, allowing life to arise on a sunless planet.

## Cosmic strings



If they exist, would be extremely thin with diameters of the same order of magnitude as that of a **proton**,  
i.e. ~ **1 fm**, or smaller



Could provide precious observational signatures of **string theory**

When two **cosmic strings** interact, the **cosmic string loop** is formed

which wiggle and oscillate, producing gravitational waves, then slowly shrink as they lose energy until they disappear



The only thing that can make a bigger atom than hydrogen is a star. The **entire periodic table**, every element you have ever heard of was processed inside the body of a star. The star then unraveled or exploded... and here we are.

**We are dead stars**

## Black hole cosmology



The Hubble radius of the observable universe is equal to its Schwarzschild radius

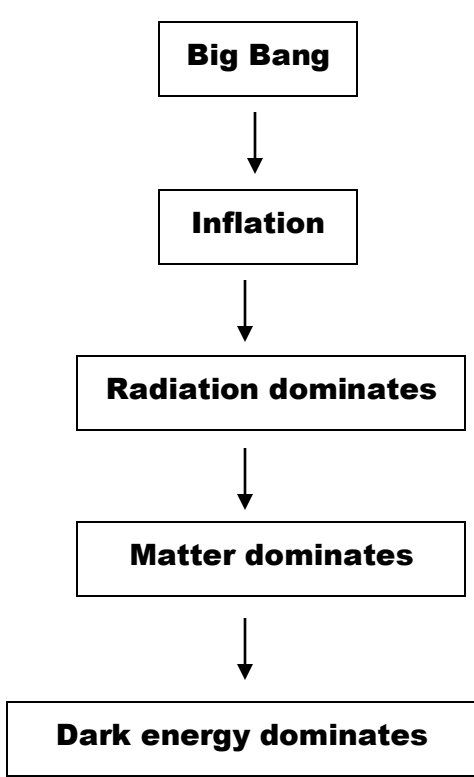
$$\frac{c}{H} = \frac{2GM}{c^2}$$

$$H = \frac{c^3}{2GM}$$

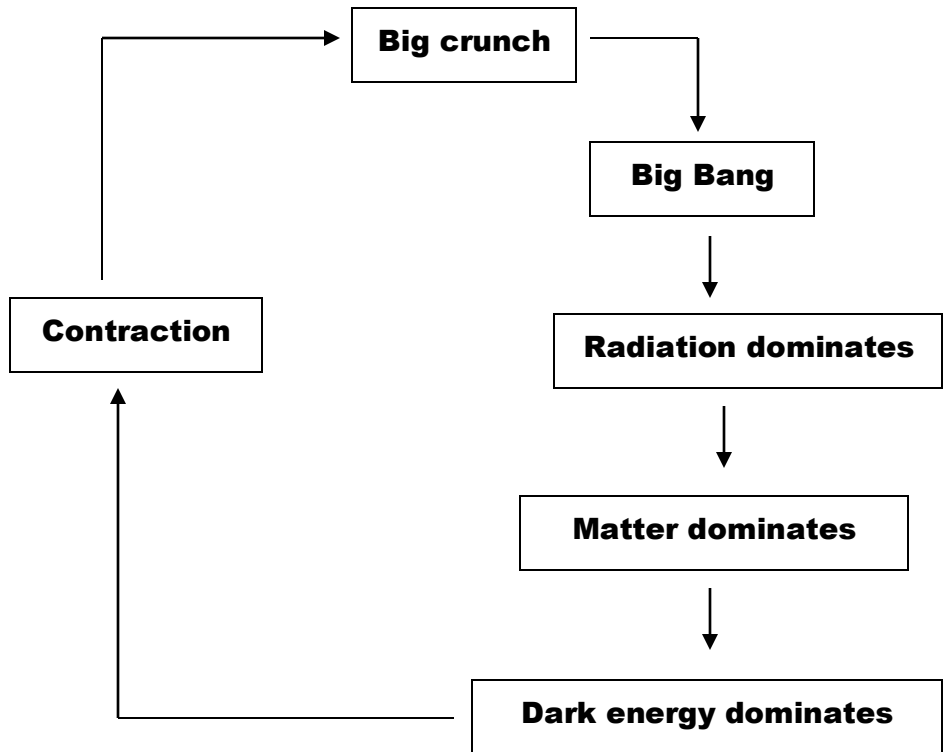
Critical density:

$$\rho_c = \frac{3H^2}{8\pi G} = \frac{3c^6}{32\pi G^3 M^2}$$

$$\rho_c \propto \frac{1}{M^2}$$



Inflationary model



Cyclic model

$$\text{Momentum} = \text{mass} \times \text{velocity}$$

Momentum is directly proportional to the particle's mass and velocity



A particle's momentum increases with increasing mass or velocity. The momentum of a large, fast moving particle is larger than that of a small, slower particle.

So Einstein was wrong when he said, "**God does not play dice.**" Consideration of black holes suggests, not only that God does play dice, but that he sometimes confuses us by throwing them where they can't be seen.

– **Stephen Hawking**

$$\text{Total charge} = \text{number of electrons} \times \text{electronic charge}$$

If we put a giant mirror 10 light years away from Earth and looked at it through a telescope, theoretically we'd see 20 years into the past.

$$Q = ne$$



$$\frac{dQ}{dt} = \frac{dn}{dt} \times e$$

$$I = \frac{dn}{dt} \times e$$

If  $I = \text{Planck current} = 3.479 \times 10^{25} \text{ A}$ :

$$\frac{dn}{dt} = \frac{I}{e} = \frac{3.479 \times 10^{25}}{1.602176634 \times 10^{-19}}$$

$$\frac{dn}{dt} = 2.171421007 \times 10^{44}$$

In **Planck current**,  $2.171421007 \times 10^{44}$  electrons are flowing every second.

|  |                          |                                 |           |
|--|--------------------------|---------------------------------|-----------|
| $KE = \frac{p^2}{2m_0}$                      | $KE = \frac{3}{2} k_B T$ | $KE = \frac{p^2}{2m_0}$         | $KE = eV$ |
| $KE = \frac{p}{2} \sqrt{\frac{3k_B T}{m_0}}$ |                          | $KE = p \sqrt{\frac{eV}{2m_0}}$ |           |

| <b>Relativistic particle</b>   | <b>Non relativistic particle</b>   |
|--|--|
| $pv_p = mc^2$ <p>where: <math>v_p = \text{phase velocity}</math></p> $(mv) v_p = mc^2$ $vv_p = c^2$ <p><b>Since <math>v &lt; c</math>:</b></p> $v_p > c$ | $pv_p = \frac{m_0 v^2}{2}$ $(m_0 v) v_p = \frac{m_0 v^2}{2}$ $v_p = \frac{v}{2}$ |

$$pv_p = mc^2$$

$$\frac{hv_p}{\lambda} = mc^2$$

$$hv = mc^2$$

$$\frac{hc}{\lambda_c} = m_0c^2$$

$$hv_c = m_0c^2$$

↓

**Compton frequency**

As the Compton frequency increases, the Compton wavelength gets shorter

$$mc^2 = \frac{m_0c^2}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$v = \frac{v_c}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{v_c}{\sqrt{1 - \frac{v}{v_p}}}$$

**The black holes of nature are the most perfect macroscopic objects there are in the universe: the only elements in their construction are our concepts of space and time.**



**-Subrahmanyan Chandrasekhar**

$$\Delta\lambda = \lambda_C (1 - \cos\theta)$$

- $\Delta\lambda$  is the Compton wavelength shift
- $\lambda_C$  is the Compton wavelength of the electron
- $\theta$  is the scattering angle

|                                       |                      |
|---------------------------------------|----------------------|
| $\Delta\lambda = 0$                   | $\theta = 0^\circ$   |
| $\Delta\lambda = \lambda_C$           | $\theta = 90^\circ$  |
| $\Delta\lambda = \frac{\lambda_C}{2}$ | $\theta = 120^\circ$ |
| $\Delta\lambda = 2\lambda_C$          | $\theta = 180^\circ$ |

The **Zeroth law of thermodynamics** is very similar to the transitive property of equality in mathematics: If  $X = Y$  and  $Y = Z$ , then  $X = Z$ .

X and Z are both in thermal equilibrium if two systems, X and Y, are in thermal equilibrium with one another and Y is in thermal equilibrium with a third system, Z.

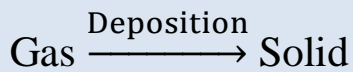
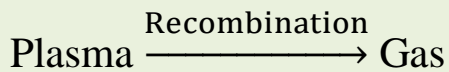
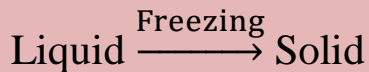
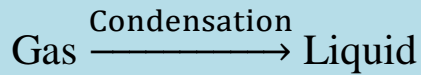
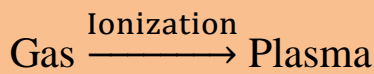
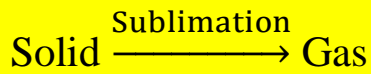
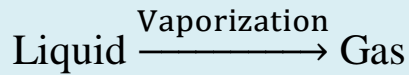
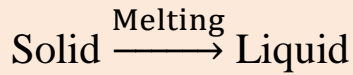
**If the planet Earth was just 5% closer to the Sun, it would experience runaway greenhouse effect – with temperatures rising to nearly 900 degrees Fahrenheit. Conversely, if the planet Earth were about 20% farther from the Sun, it would experience runaway glaciations of the kind that has left Mars sterile.**

|  |  |
|--|--|
| <b>Law of conservation of mass</b>     | mass can neither be created nor destroyed but is transformed from one form to another  |
| <b>Law of conservation of energy</b>   | energy can neither be created nor destroyed but is transformed from one form to another  |
| <b>Law of conservation of momentum</b> | momentum is neither created nor destroyed but only changed through the action of forces as described by <b>Newton's laws of motion</b> |

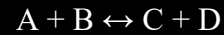
### Computable Universe Theory

Our external physical reality is defined by **computable functions**

(The **functions** that can be calculated using a mechanical calculation device given unlimited amounts of time and storage space)



**For the reversible reaction:**



the equilibrium constant is :

$$K = \frac{\overbrace{[C][D]}^{\text{PRODUCTS}}}{\underbrace{[A][B]}_{\text{REACTANTS}}}$$

- If  $K > 1$  then equilibrium favors the formation of the products
- If  $K < 1$  then equilibrium favors the formation of the reactants
- If  $K = 1$  then equilibrium neither favors the formation of the products nor the reactants

The typical radius of an **atomic nucleus** is  $10^{-15}$  m, which is a factor  $\sim 10^{20}$  greater

than the **Planck length**  $\sqrt{\frac{\hbar G}{c^3}}$ .

**Special Relativity**



A theory of space-time

(Deals with inertial frame of reference)

**General Relativity**



A theory of gravity

(Deals with non-inertial frame of reference)

- Like poles of magnets repel each other and unlike poles attract each other
- Like charges repel each other and unlike charges attract each other



Assuming the change of entropy  $\Delta S$  near the holographic screen to be linear in the displacement  $\Delta x$  of a test particle, the entropic force  $F$  acting on the particle is given by the relation:

$$F = \frac{T \times \Delta S}{\Delta x}$$

where  $T$  is the temperature. Taking  $T$  to be given by Unruh's temperature:

$$F = \frac{\Delta S}{\Delta x} \times \frac{\hbar a}{2\pi k_B c}$$

where  $a$  stands for the acceleration of the particle. Since  $F = ma$ :

$$\Delta S = 2\pi k_B \frac{\hbar}{mc} \Delta x$$

where:  $m$  is the mass of the particle.

$$KE = \gamma m_0 c^2 - m_0 c^2$$

If  $KE = m_0 c^2$ :

$$\gamma = 2 \longrightarrow$$

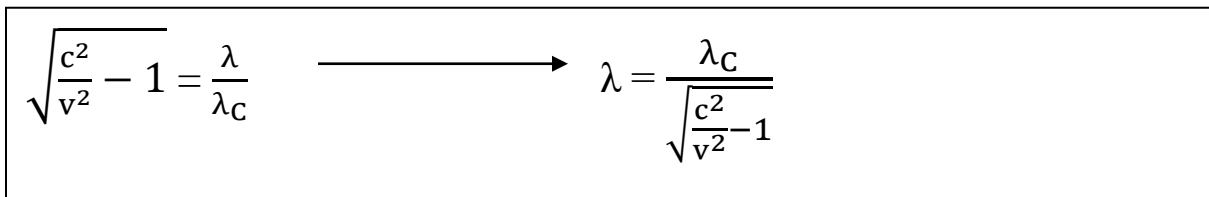
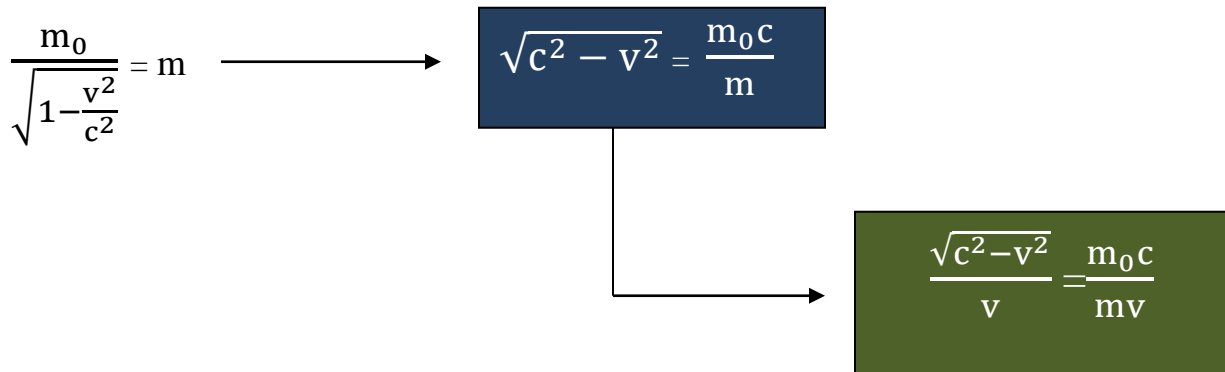
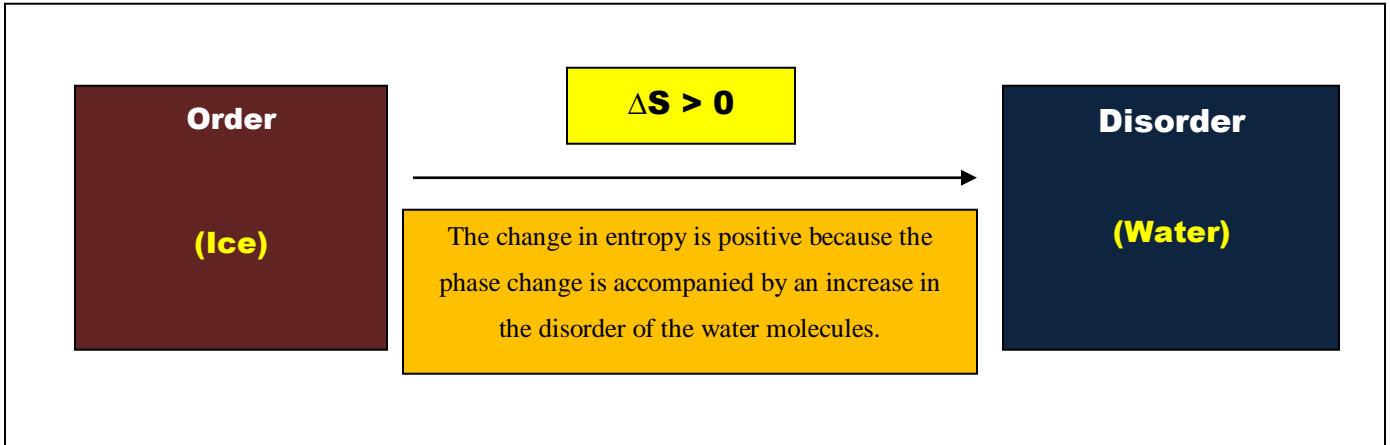
$$\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = 2$$

$$v = \sqrt{\frac{3}{4}} c = 0.86602540378 c$$

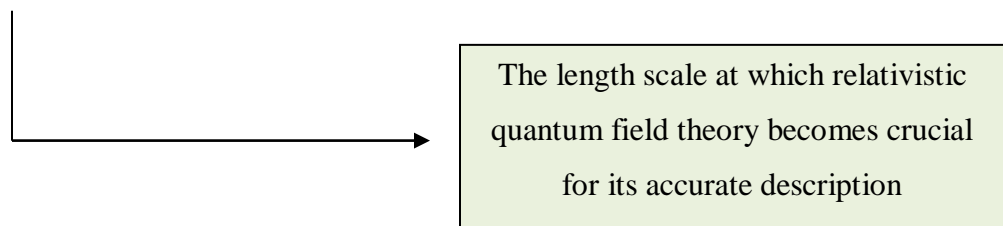
So as the particle travels at velocity  $v = 0.86602540378 c$ , the kinetic energy of the particle equals its rest mass energy.

3 distinct ways to transport heat are:

- **Conduction** → Heat transfer through direct physical contact
- **Convection** → Heat transfer by the movement of a fluid
- **Radiation** → A form of heat transfer that takes place when electromagnetic radiation is emitted or absorbed




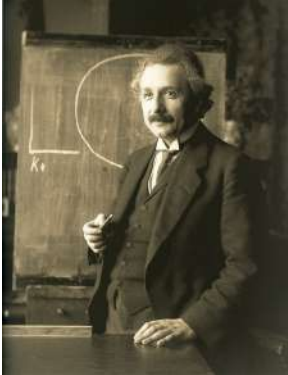
$\lambda_C$  → **Compton Wavelength of the particle**



If  $\lambda = \lambda_C$ :

$$v = \frac{c}{\sqrt{2}} = 0.70710678118 c$$

If a particle travels at a velocity  $v = 0.70710678118 c$ , then its wavelength  $\frac{h}{mv}$  will be equal to its Compton wavelength  $\frac{h}{m_0 c}$

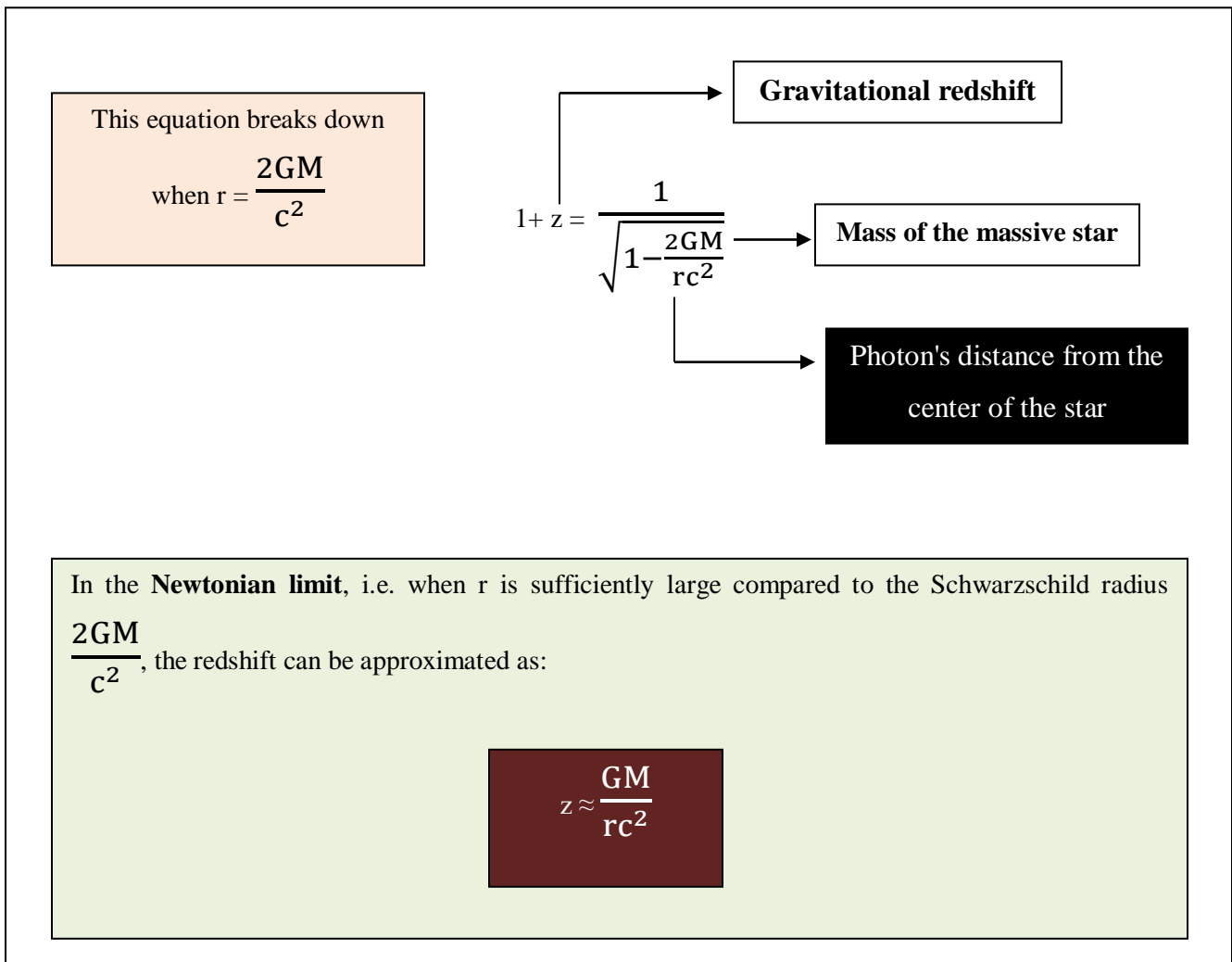
| <b>Compton effect</b>  | <b>Photoelectric effect</b>  |
|--|--|
| <p>The frequency of the scattered photon is lower than that of the incident photon.</p>  <p><i>Arthur H. Compton</i></p> <p><b>Arthur Compton explained this effect</b></p> | <p>The frequency is not observed as the photon disappears after interacting with the electrons.</p>  <p><i>Albert Einstein</i></p> <p><b>Albert Einstein explained this effect</b></p> |

$$\% \text{ mass increase} = \frac{m - m_0}{m_0} \times 100$$

$$\% \text{ mass increase} = (\gamma - 1) \times 100$$

The **Lorentz factor** is always more than 1, but as a particle's velocity approaches the speed of light, it approaches infinity. There are no relativistic effects when  $\gamma$  is 1.

| Force            | Carrier Particle                                  | Range          |
|------------------|---|----------------|
| Gravity          | Graviton (theorized)                              | $\infty$       |
| Weak             | W and Z bosons                                    | $< 10^{-18}$ m |
| Electromagnetism | Photon  | $\infty$       |
| Strong           | Pi mesons or pions ( <b>now known as gluons</b> ) | $< 10^{-15}$ m |



Every particle in the cosmos, **according to string theory**, can be classified into two groups: fermions (**which are matter particles like electrons**) or bosons (**which are force carrier particles like photons**)

- As planets can form around low mass stars, **low mass stars** are essential for the existence of life.
- **High-mass stars** are essential for life because high-mass stars can generate heavy elements such as carbon, oxygen, nitrogen, etc by nuclear fusion and release the created elements into space by Type II supernova explosions.

**Without both, life is impossible!**

### The Third Law of Thermodynamics



A perfect crystal at zero Kelvin has zero entropy

### Chandrasekhar limit:

$$M_{\text{limit}} = \frac{\omega_3^0 \sqrt{3\pi}}{2} \times \frac{1}{\mu_e} \times \frac{(\text{Planck mass})^3}{(\text{proton mass})^2} \approx 1.4M_{\text{sun}}$$

- $\mu_e$  is the average molecular weight per electron, which depends upon the chemical composition of the star.
- $\omega_3^0 \approx 2.018236$  is a constant connected with the solution to the Lane–Emden equation

**The maximum mass of a stable white dwarf star**

The white dwarf remains the same forever if its mass is within the limit. The mass of the star that exceeds the limit will explode and become a supernova.

## Schönberg–Chandrasekhar limit:

$$\frac{M_{\text{core}}}{M} \approx 0.37 \frac{\mu_{\text{env}}^2}{\mu_{\text{core}}^2}$$

An upper limit on a main-sequence star's core mass: If this limit is crossed, a star will exit the main sequence and turn into a red giant when its hydrogen reserves run out, which accounts for about 12% of its initial mass.

- $M_{\text{core}}$  → mass of the core.
- $M$  → mass of the whole star.
- $\mu_{\text{core}}$  → mean molecular mass of the core.
- $\mu_{\text{env}}$  → mean molecular mass of the envelope.

**According to Albert Einstein's theory of Relativity,  
the farther you are from the earth's surface, the  
faster time passes.**

For stars:

$$t_{\text{dyn}} < t_{\text{th}} < t_{\text{nuc}}$$

- $t_{\text{dyn}}$  = timescale of collapsing star, e.g. supernova
- $t_{\text{th}}$  = timescale of star before nuclear fusion starts, e.g. pre-main sequence lifetime
- $t_{\text{nuc}}$  = timescale of star during nuclear fusion, i.e. main-sequence lifetime

**1 becquerel = 1 Bq = 1 decay per second.**

**1 curie = 1 Ci =  $3.7 \times 10^{10}$  Bq**

### Bremmermann's limit

$$\left\{ \frac{c^2}{h} \approx 1.36 \times 10^{50} \text{ bits per second per kilogram} \right\}$$



The maximum rate of computation that can be achieved in a self-contained system in the material universe

### Abundance Ratio



The logarithm of the ratio of two metallic elements in a star relative to their ratio in the Sun. For example, the logarithm of the magnesium to iron ratio in a star compared to the magnesium to iron ratio in the Sun is used to describe **the abundance ratio of magnesium to iron**.

### Did you Know?

A star's total radiation output across all wavelengths of the electromagnetic spectrum is measured by the bolometric magnitude of the star.

**The apparent magnitude of the faintest object that can be seen with the unaided eye or via a telescope is the limiting magnitude.**

**If earth stops rotating suddenly and you're not belted to earth's surface, humans would suddenly fly towards east at 464.9216 m/s at the equator and die instantly.**

About ten billion years into its life, the **Milky Way galaxy** has now converted about 90% of its initial gas content into stars.

- **Conformal cyclic cosmology:** The universe goes through infinite endless cycles from creation to destruction over and over again.
- **Loop quantum cosmology:** Application of **loop quantum gravity** to eliminate singularities such as the big bang and big crunch singularity.

**Eternal recurrence:** The idea that all events in the world repeat themselves in the same sequence through an eternal series of cycles

### **Quantum emergence**

**Space and time develop out of a primeval state described by a quantum theory of gravity**

If the **Andromeda Galaxy** were visible to the naked eye, it would be six times larger than the moon in the night sky.



## Phantom energy



Dark energy with  $\frac{\text{its pressure}}{\text{its energy density}} < -1$



Causes a **Cosmic Doomsday**

The matter of the universe (**from stars and galaxies to atoms and subatomic particles**) and even spacetime itself is progressively torn apart by the expansion of the universe at a certain time in the future – until distances between particles will become infinite.

- Dark energy with  $\frac{\text{its pressure}}{\text{its energy density}} < -\frac{1}{3} \rightarrow$  **cosmic acceleration**
- If the **expansion is accelerating**:

$$-1 < \text{Dark energy with } \frac{\text{its pressure}}{\text{its energy density}} < -\frac{1}{3}$$

Then the scale factor of the universe grows more rapidly than the horizon. Galaxies disappear beyond the horizon.

|  |           |
|--|-----------|
| The number of atoms in the body of an average male ( <b>60 kg</b> ) is about | $10^{27}$ |
| The number of atoms making up the earth is about                             | $10^{50}$ |
| The number of atoms in the known universe is estimated at                    | $10^{80}$ |

|                             |   |
|-----------------------------|---|
| <b>Isoenthalpic process</b> | $\Delta H = 0$ (Enthalpy constant)        |
| <b>Isoentropic process</b>  | $\Delta S = 0$ (Entropy constant)         |
| <b>Steady state process</b> | $\Delta U = 0$ (Internal energy constant) |

**Weakless universe**



A hypothetical universe that contains no weak interactions

**General relativity**

**Clocks run slower in deeper gravitational wells**

**Rays of light bend in the presence of a gravitational field**

**Rotating masses drag along the spacetime around them**

The universe is expanding, and the far parts of it are moving away from us faster than the speed of light

**The properties of planetary orbits are dependent on spacetime structure**

## Cosmic neutrino background ( $T_\nu = 1.95\text{K}$ )

(Low-energy neutrinos left over from early universe)



whose thermal energy  $\frac{3k_B T_\nu}{2}$  falls below its rest mass energy  $m_\nu c^2$

### Redshift

Photons, which have no mass, lengthen their wavelength rather than slow their speed

### Dark matter

#### Hot dark matter

- possess a very small mass
- moves at ultrarelativistic speed

#### Warm dark matter

According to **physicists**, Dark Matter binds the universe (**It interacts with gravity**), whereas the Dark Energy loosens up or speed up the expansion of the universe (**It creates space**).

- possess mass between **Cold dark matter** and **Hot dark matter**
- moves at speeds perhaps thousands of miles per hour

#### Cold dark matter

- more massive
- moves slowly compared to the speed of light

|                                       |  |
|---------------------------------------|--|
| <b>decoupling</b>                     | some type of particle stops being destroyed as often as it is created  |
| <b>Compton scattering</b>             | photon collision with a charged particle that emits a different photon   |
| <b>baryon acoustic oscillations</b>   | density fluctuations stemming from acoustic waves in the early universe  |
| <b>Cosmic Background Imager</b>       | radio telescope in Chile ( <b>a country in western South America</b> ) aimed at studying the cosmic microwave background (CMB)         |
| <b>C-Band All Sky Survey</b>          | 6 cm survey of the entire sky  |
| <b>CMB anisotropies</b>               | variations in the spectrum of the CMB  |
| <b>Hubble expansion</b>               | ongoing expansion of the universe  |
| <b>CMB lensing</b>                    | gravitational lensing of the CMB   |
| <b>Cosmic Background Explorer</b>     | 1989 satellite for surveying the CMB   |
| <b>cosmological simulation</b>        | simulation of the history of the universe  |
| <b>dark energy</b>                    | energy theorized to accelerate the expansion of the universe   |
| <b>dark matter</b>                    | non-baryonic matter  |
| <b>ordinary matter</b>                | baryonic matter  |
| <b>DASI</b>                           | <b>Degree Angular Scale Interferometer</b><br>(2000–2003 South Pole CMB telescope)   |
| <b>DGP gravity</b>                    | alternative theory of gravity addressing " <b>dark energy</b> "  |
| <b>extragalactic background light</b> | electromagnetic radiation from star formation and active galactic nucleus  |
| <b>epoch of reionization</b>          | time when the universe's neutral hydrogen atoms ionized  |
| <b>Galactic Emission Mapping</b>      | radio sky survey   |
| <b>gravitational wave background</b>  | combination of weak gravitational waves such as from distant source  |
| <b>inflation</b>                      | theorized rapid expansion of the <b>very early universe</b> by 78 orders-of-magnitude within its first $10^{-32}$ seconds of existence |
| <b>initial fluctuations</b>           | early universe density variations leading to the large scale structure of the current universe   |
| <b>initial fluctuation spectrum</b>   | the power spectral density of the early universe   |
| <b>intensity mapping</b>              | measure of cosmic matter density   |

|                                       |  |
|---------------------------------------|--|
| <b>Isotropy</b>                       | uniformity in all directions   |
| <b>LAMBDA</b>                         | <b>Legacy Archive for Microwave Background Data Analysis</b><br>(NASA archive center for CMB data)   |
| <b>light cone</b>                     | the region of space-time from which light can reach you  |
| <b>LiteBIRD</b>                       | <b>Light satellite for the studies of B-mode polarization and inflation from cosmic background radiation detection</b><br>(planned spacecraft to map CMB polarization) |
| <b>magnetic energy spectrum</b>       | spatial spectral density of the energy associated with magnetic fields   |
| <b>microwave</b>                      | electromagnetic radiation (frequency 300 MHz to 300 GHz)   |
| <b>millimeter astronomy</b>           | observation and analysis at wavelengths on the order of a millimeter   |
| <b>maximum likelihood mapmaking</b>   | method of making maps using varying data   |
| <b>Planck</b>                         | microwave telescope in space   |
| <b>polarimetry</b>                    | measurement of polarization of electromagnetic radiation   |
| <b>polarization</b>                   | property of waves that oscillate in a particular direction   |
| <b>primordial gravitational waves</b> | gravitational waves formed in the early universe   |
| <b>QMAP</b>                           | mid 1990s balloon-based CMB survey   |
| <b>QUIJOTE</b>                        | survey to measure the polarization of the CMB  |
| <b>Rayleigh scattering</b>            | wave scattering by particles smaller than the wavelength   |
| <b>recombination</b>                  | time when the universe's ionized hydrogen atoms neutralized  |
| <b>relic</b>                          | particles left over from the events of the early universe  |
| <b>spinning dust emission</b>         | microwave emission from spinning dust particles that carry a dipole electric field   |
| <b>electric dipole radiation</b>      | electromagnetic radiation generated by an oscillating electric dipole  |
| <b>surface of last scattering</b>     | sphere around us from which CMB photons are just reaching us   |
| <b>Sachs-Wolfe effect</b>             | gravitational redshift of the CMB  |
| <b>thermal dust emission</b>          | glow of dust heated by stars and active galactic nucleus   |
| <b>Topological defects</b>            | stable configurations of matter formed at phase transitions in the very early Universe   |

**68% of the universe is dark energy and 27% is dark matter. Both of these are invisible, even with a telescope, which means we are only able to see 5% of the universe.**

**Pistol star** is one of the most luminous known stars in our galaxy. It radiates the same amount of energy in just 20 seconds as our sun does in an entire earth year

What is a star life cycle? A star life cycle is birth, **Nebula** then to its death. A star is born from the death of another star.

Science merely amplifies the capabilities of human beings. Science gives us the ability to do ill and to do good more than we had, and to question science in this respect is like questioning whether people ought to have two hands or just one, because with two hands they could do more evil than they can with just one.

**- Steven Weinberg**

**Avogadro's hypothesis** states that equal volumes of gases at the same temperature and pressure contain the same number of particles. At **Standard Temperature and Pressure**, one mole ( $6.02 \times 10^{23}$  particles) of any gas occupies a volume of 22.4 liters

|                              |  |
|------------------------------|--|
| <b>Bernoulli's principle</b> | As speed of the fluid increases, pressure decreases. |
|------------------------------|--|

**A neutron star known as PSR 11748-2446AD, has an equator spinning at 24% the speed of light, which translates to over 70,000 kilometers per second.**

| <b>Pauli exclusion principle</b>                                | <b>Fermions</b> | <b>Bosons</b> |
|---|-----------------|---------------|
| prohibits two particles from existing in the same quantum state | ✓               | ✗             |

We can have a number of bosons coexisting in a single energy level.

**Greisen-Zatsepin-Kuzmin limit**



**Theoretical limit on the energy of cosmic ray protons** traveling from other galaxies through the intergalactic medium to our galaxy

**$5 \times 10^{19}$  eV or about 8 joules**

(The energy of a proton travelling at  $\approx 99.999999999999999998\%$  the speed of light)

|                           |  |
|---------------------------|--|
| <b>Isothermal process</b> | $\Delta T = 0$ ( <b>Temperature constant</b> )                                 |
| <b>Isobaric process</b>   | $\Delta P = 0$ ( <b>Pressure constant</b> )                                    |
| <b>Isochoric process</b>  | $\Delta V = 0$ ( <b>Volume constant</b> )                                      |
| <b>Adiabatic process</b>  | $\Delta Q = 0$ ( <b>No heat flow between the system and the surroundings</b> ) |

$$\Delta G = \Delta H - T \Delta S$$

- If  $\Delta G$  is negative ( $< 0$ ), the process is **spontaneous** (exergonic).
- If  $\Delta G$  is positive ( $> 0$ ), the process is **non spontaneous** (endergonic).

|                                     | <b><math>\Delta H &lt; 0</math></b>                 | <b><math>\Delta H &gt; 0</math></b>                  |
|-------------------------------------|---|--|
| <b><math>\Delta S &gt; 0</math></b> | Spontaneous at all T<br>( $\Delta G < 0$ )          | Spontaneous at high T<br>(when $T\Delta S$ is large) |
| <b><math>\Delta S &lt; 0</math></b> | Spontaneous at low T<br>(when $T\Delta S$ is small) | Non Spontaneous at all T<br>( $\Delta G > 0$ )       |

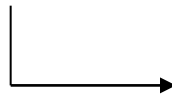
**The universe has no centre and is constantly expanding every second - making it impossible to reach the edge**

- The universe is so vast that when you look into the night sky, you're looking back in time.
- At 25 km tall and 600 km wide, **Olympus Mons** on mars is the tallest mountain on any of the planets of the solar system. That's 3 times the size of Everest.
- **Sagittarius B** is a vast molecular cloud of gas and dust floating near the centre of the Milky Way. It contains 10-billion-billion-billion liters of alcohol.
- There are probably more than **170 billion galaxies** in the observable universe.



The orbit of Neptune's moon **Triton** is slowly decaying, meaning it is getting closer to the planet. In 3.6 billion years time Triton will drop below the **Roche limit**, and Neptune's tidal forces will tear Triton apart, forming a ring around the planet

$$\frac{\sin i}{\sin r} = n$$



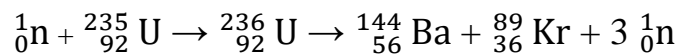
The relation between angle of incidence and angle of refraction depends upon the refractive index of the medium. The **refracted angle** is greater than the **incidence angle** when a light ray travels from a denser medium to a rarer medium and vice versa.

The **wave nature of light** was demonstrated convincingly for the first time in 1801 by Thomas Young by a wonderfully simple experiment. He let a ray of sunlight into a dark room, placed a dark screen in front of it, pierced with two small pinholes, and beyond this, at some distance, a white screen. He then saw two darkish lines at both sides of a bright line, which gave him sufficient encouragement to repeat the experiment, this time with spirit flame as light source, with a little salt in it to produce the bright yellow sodium light. This time he saw a number of dark lines, regularly spaced; the first clear proof that light added to light can produce darkness. This phenomenon is called **interference**. **Thomas Young** had expected it because he believed in the wave theory of light.

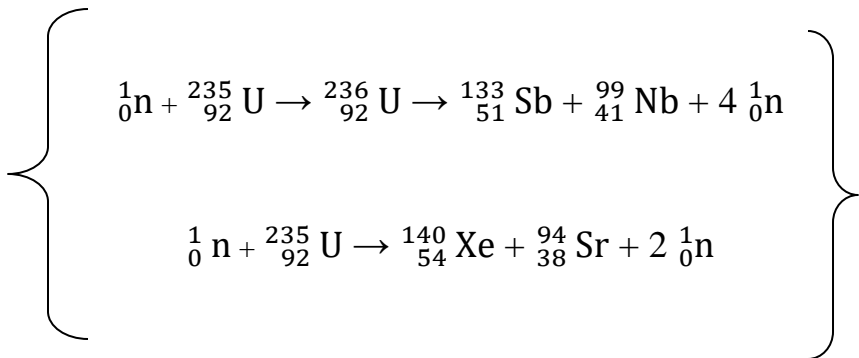
- **Dennis Gabor**

|                           | <b>Interference</b> | <b>Diffraction</b> | <b>Polarization</b> |
|---------------------------|---------------------|--------------------|---------------------|
| <b>Longitudinal waves</b> | ✓                   | ✓                  | ✗                   |

**Longitudinal waves** cannot be polarized because their particles vibrate in the same direction that the wave travels



**Nuclear Fission does not always produce barium and krypton.**



${}^{16}_8\text{O}$  atom has 8 neutrons, 8 protons and 8 electrons.

- Mass of 8 neutrons =  $8 \times 1.00866 \text{ u}$
- Mass of 8 protons =  $8 \times 1.00727 \text{ u}$
- Mass of 8 electrons =  $8 \times 0.00055 \text{ u}$

If one wants to break the oxygen atom into 8 protons, 8 neutrons and 8 electrons, this energy has to be supplied.

The expected mass of  ${}^{16}_8\text{O}$  atom =  $8 \times 2.01593 \text{ u} = 16.12744 \text{ u}$ .

The observed mass of  ${}^{16}_8\text{O}$  atom =  $15.99493 \text{ u}$

The mass of the  ${}^{16}_8\text{O}$  atom is less than the total mass of its constituents by **0.13251u**.

Binding energy of  ${}^{16}_8\text{O}$  atom = **0.13251u**  $\times c^2$



**Every star in the night sky is larger than the sun.**

Even if you were able to travel close to the speed of light (**186000 miles per second**) you would still take 100000 years to cross the Milky Way galaxy.

**Hypervelocity stars** are stars that have been ejected from their systems and sent rocketing through intergalactic space at speeds up to 2 million miles per hour. Most of the hypervelocity stars that have been identified so far are of similar size and mass as the Sun, but theoretically could be bigger.

- Reynolds number =  $\frac{\text{inertial force}}{\text{viscous force}}$

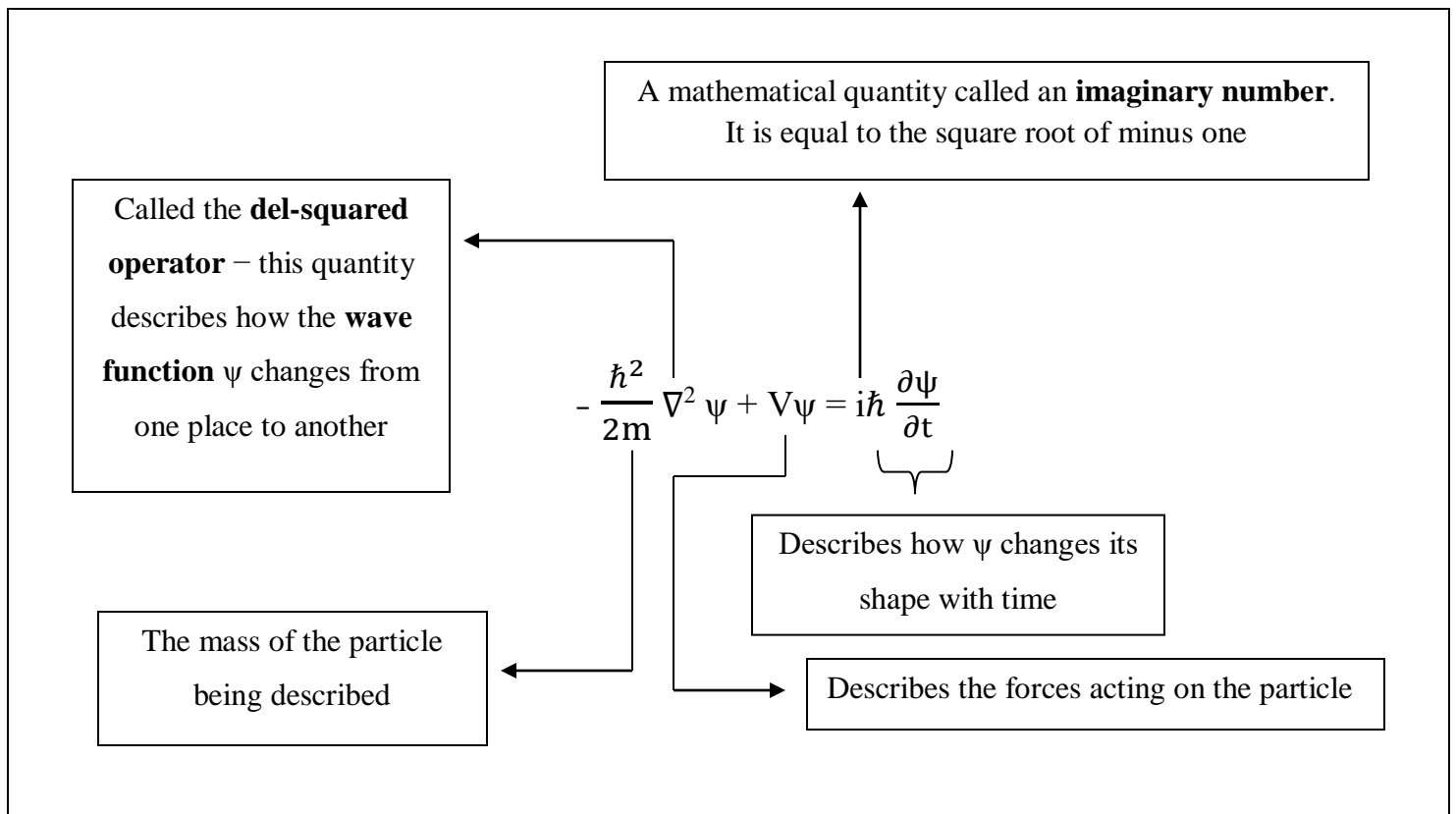
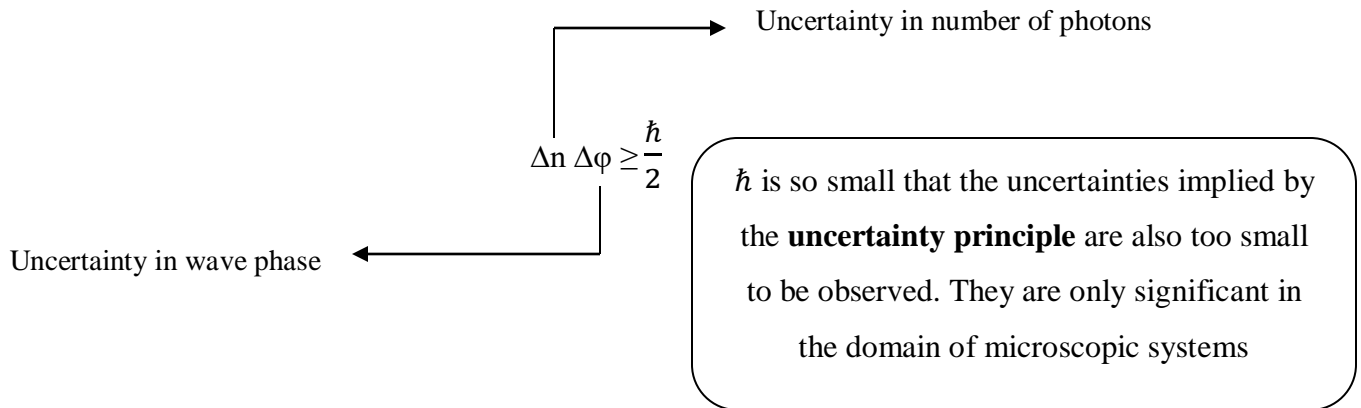
- Froude number =  $\frac{\text{inertial force}}{\text{gravitational force}}$

- Rossby number =  $\frac{\text{inertial force}}{\text{Coriolis force}}$

- Prandtl number =  $\frac{\text{momentum diffusivity}}{\text{thermal diffusivity}}$

- Mach number =  $\frac{\text{speed of flow}}{\text{sound speed}}$

**Numbers that describe the flow characteristics of a fluid**



- The largest known black hole (**Holmberg 15A**) has a diameter of 1 trillion km, more than 190 times the distance from the Sun to Pluto.

- The **coldest known natural place** in the universe is the **Boomerang Nebula**. At  $-272.15^{\circ}\text{C}$  it is  $1^{\circ}\text{C}$  warmer than absolute zero, and  $2^{\circ}\text{C}$  colder than background radiation from the hot **Big Bang**.

Since:

$$e = 1.602192 \times 10^{-19} \text{ C}$$

**There are about  $6 \times 10^{18}$  electrons in a charge of  $-1\text{C}$**

The accelerated motion of charge  $q_1$  produces electromagnetic waves, which then propagate with the speed  $c$  ( $3 \times 10^8 \text{ m/s}$ ), reach  $q_2$  and cause a force on  $q_2$

$$\text{Force on } q_2 \text{ due to } q_1 = \frac{q_1 q_2}{4\pi\epsilon_0 r^2}$$

$$q_1 = q_2 = 1\text{C}, r = 1\text{m}$$

$$\text{Force on } q_2 \text{ due to } q_1 = 8.99 \times 10^9 \text{ N}$$

- If  $q_1$  and  $q_2$  are of opposite signs, the force between the charges is attractive.
- If  $q_1$  and  $q_2$  have the same signs, the force between the charges is repulsive.

**Superposition Principle**



The force with which two charges attract or repel each other are not affected by the presence of a third charge

**A small percentage of the static shown by our Television is the Big Bang's afterglow.**

### Wave equation:

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2} c^2$$

The **Fourier transform** is essential to understanding more complex wave structures like **human speech**

**Describes the behavior of waves** — a vibrating guitar string, ripples in a pond after a stone is thrown, or light coming out of an incandescent bulb.

### Schrödinger's equation:

$$i\hbar \frac{\partial}{\partial t} \psi = H\psi \text{ (time dependent)}$$

$$E \psi = H\psi \text{ (time independent)}$$

Describes the energy and position of the **electron** in space and time – taking into account the **matter wave nature** of the electron inside an atom

- Astronauts can grow up to 3% taller in outer space, or about 2 inches for a 6-foot-tall person
- **According to quantum mechanics**, every time you get confused between two things and decide to choose to go with one of them, the universe splits into two parts, where you choose the other option in another universe.
- In 2012, particles smashed together in the Large Hadron Collider's 27-kilometer circular tunnel conjured up the **Higgs boson** — the last missing particle predicted by **the Standard Model of particle physics**. The Higgs particle is supposed to be responsible for giving particles their masses (this particle would answer the basic question of why particles have the masses they do—or indeed why they have any mass at all).

**Weight = mass × acceleration due to gravity**



On Earth, it generally takes the value =  $9.81 \text{ m/s}^2$  however on other planets and at extreme heights on Earth, it take other values.

An object on the Moon would have the same mass as an identical object on Earth, but its weight would be less since the Moon's gravitational field is weaker

**Coefficient of restitution of the collision:**

$$e = \frac{\text{speed of separation of the particles}}{\text{speed of approach of the particles}}$$

- $e = 1$  (**elastic collision**)

→ speed of separation of the particles = speed of approach of the particles

- $0 < e < 1$  (**inelastic collision**)

→ speed of separation of the particles is not zero but less than speed of approach of the particles

- $e = 0$  (**perfectly inelastic collision**)

→ colliding particles do not separate out but move with same speed

**1 Second to Get the Moon**

**8 Minutes to Get the Sun**

**2000 years to get out of Milky Way**

**46.5 Billion Years to Get the Edge of the Observable Universe**

- **Coronal loops** are structured arcs of glowing, electrified plasma that flow along the powerful, curved, magnetic fields above the Sun's surface. This one is roughly **4 times** the size of Earth.
- A new star is born in our galaxy roughly every 18 day.
- Stars twinkle because we see them through the **wafting of the atmosphere**.

**The color and size of a star tells astronomers its age.**

- Yellow dwarfs and blue giants are young.
- Red giants and red supergiants are older.
- White dwarfs and black dwarfs are the oldest.

**pH scale**

| Strong acids |   | ← More acidic → |   |   | Weak acids |   | Neutral | Weak bases |   | ← More basic → |    |    | Strong bases |    |
|--------------|---|-----------------|---|---|------------|---|---------|------------|---|----------------|----|----|--------------|----|
| 0            | 1 | 2               | 3 | 4 | 5          | 6 | 7       | 8          | 9 | 10             | 11 | 12 | 13           | 14 |

Most constellations are constantly shifting in the sky. However, there are some constellations that never set or rise. These constellations are known as **circumpolar constellations**. These are the constellations that make excellent reference points for locating **seasonal constellations**.



The **Apollo astronauts'** footprints on the moon will probably stay there for at least 100 million years

**It would take light 650 years to travel from one end of the Tarantula Nebula to the other.**

The uniform character of mathematics is the essence of science, for mathematics is the foundation of all exact scientific knowledge.

**DAVID HILBERT, 1862 – 1943**

Geometry is one and eternal shining in the mind of God. That men share in it is among the reasons that Man is the image of God.

**JOHANNES KEPLER, 1571 – 1630**

Mathematics is the language in which the gods speak to people.

**PLATO, C. 427 BC – C. 347 BC**

- Nebulae are **interstellar clouds** that are made of plasma, helium, hydrogen and dust
- When nuclear fusion in a star much like our Sun ceases, the star dies. It expels its outer gaseous layers and leaves behind a tiny, hot, dense core called a **white dwarf**. The cosmic result – a **planetary nebula**.
- If two pieces of the same type of metal touch in space, they will bond and be permanently stuck together. This amazing effect is called **cold welding**.

In my experience most mathematicians are intellectually lazy and especially dislike reading experimental papers.

**FRANCIS CRICK, 1916 – 2004**

The legend that every cipher is breakable is of course absurd, though still widespread among people who should know better.

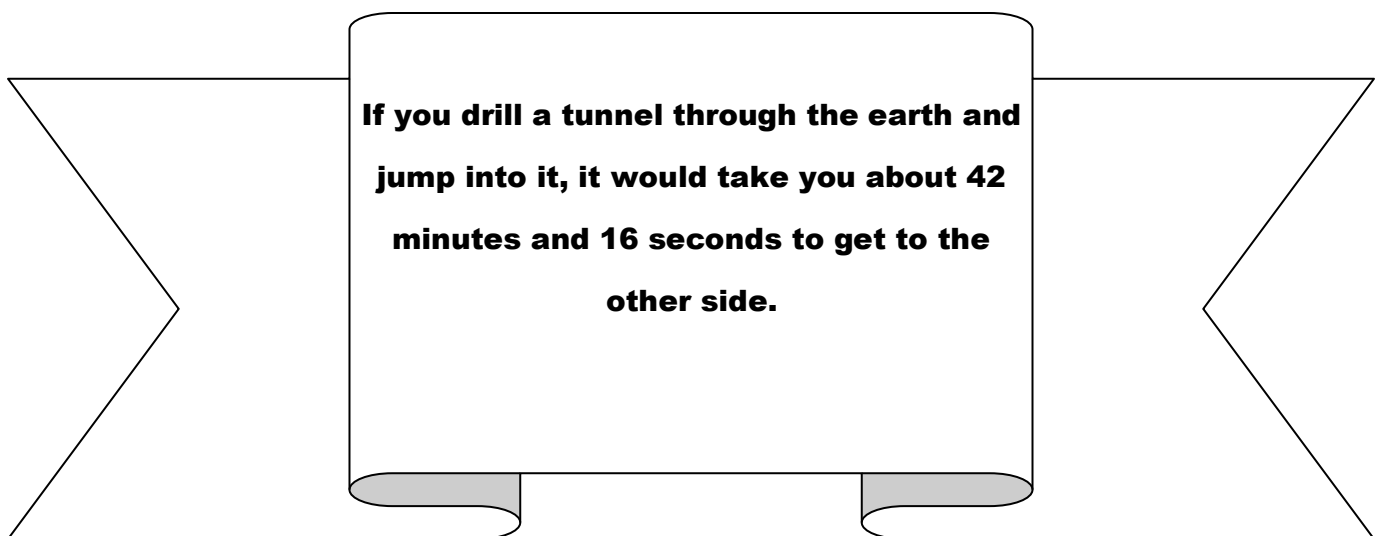
**J.E. LITTLEWOOD, 1885 – 1977**

Surely it is not knowledge, but learning; not owning but earning; not being there, but getting there; that gives us the greatest pleasure.

**CARL FRIEDRICH GAUSS, 1777 – 1855**

Actually, everything that can be known has a number; for it is impossible to grasp anything with the mind or to recognize it without this.

**PHILOLAUS, C. 470 – C. 385 BC**



Euclid's work ought to have been any educationist's nightmare... it never offers any "motivations," it has no illuminating "asides," it does not attempt to make anything "intuitive," and it avoids "applications" to a fault. It is so "humorless" in its mathematical purism that... ..it should have been spurned by students and "progressive" teachers in every generation. But it nevertheless survived intact all the turmoils, ravages, and illiteracies of the dissolving Roman Empire, of the early Dark Ages, of the Crusades, and of the plagues and famines of the later Middle Ages.

**SALOMON BOCHNER, 1899 – 1982**

- **Quantum Physics** came from the Vedas. Schrodinger, Albert Einstein and Nikola Tesla were **Vedantists**.
- **The Sun** is actually white, it just appears yellow to us through the **Earth's atmosphere**.

There may be babblers, wholly ignorant of mathematics, who dare to condemn my hypothesis, upon the authority of some part of the Bible twisted to suit their purpose. I value them not, and scorn their unfounded judgment.

**NICOLAUS COPERNICUS, 1473 – 1543**

Before the Copernican revolution, it was natural to suppose that God's purposes were specifically concerned with the earth, but now this has become an unpalatable hypothesis. If it is the purpose of the Cosmos to evolve mind, we must regard it as rather incompetent in having produced so little in such a long time.

**BERTRAND RUSSELL, 1872 – 1970**

**Quantum physics** says reality changes with observation.

**Quantum Bayesianism** says reality is observation.

Instead of particles choosing a state when observed, maybe  
observers are just updating their knowledge.

- **Quark matter** is an extremely dense phase of matter made up of subatomic particles called **quarks**. This theoretical phase would occur at extremely high temperatures and densities. It may exist at the heart of neutron stars. It can also be created for brief moments in particle colliders on Earth, such as **CERN's Large Hadron Collider**.
- Like a **black hole**, a white hole is a prediction of Albert Einstein's theory of general relativity. It is essentially a black hole in reverse: if nothing can escape from a black hole's event horizon, then nothing can enter a white hole's event horizon.

I see some parallels between the shifts of fashion in mathematics and in music. In music, the popular new styles of jazz and rock became fashionable a little earlier than the new mathematical styles of chaos and complexity theory. Jazz and rock were long despised by classical musicians, but have emerged as art-forms more accessible than classical music to a wide section of the public. Jazz and rock are no longer to be despised as passing fads. Neither are chaos and complexity theory. But still, classical music and classical mathematics are not dead. Mozart lives, and so does Euler. When the wheel of fashion turns once more, quantum mechanics and hard analysis will once again be in style.

**FREEMAN DYSON, B. 1923**

Simple laws can very well describe complex structures. The miracle is not the complexity of our world,  
but the simplicity of the equations describing that complexity.

**SANDER BAIS, B. 1945**

I am not insensible of the advantage which accrues to Applied Mathematics from the co-operation of the Pure Mathematician, and this co-operation is not infrequently called forth by the very imperfections of writers on Applied Mathematics.

**RONALD FISHER, 1890 – 1962**

- Rogue planets are not bound by any star and so "**free-float**" through Space.
- In April 2010, radio astronomers monitoring the **galaxy M82** spotted and unknown object that was moving 4 times faster than the speed of light and sending radio signals that had never been seen in the universe before.
- The average distance between asteroids in space is over **100,000 miles**, meaning an asteroid field would be very simple to navigate.
- The complex nuclear contents of a **neutron star** are all named after Italian food.

In practical applications we are concerned only with comparatively small numbers; only stellar astronomy and atomic physics deal with 'large' numbers, and they have very little more practical importance, as yet, than the most abstract pure mathematics.

**G. H. HARDY, 1877 – 1947**

Geometry, inasmuch as it is concerned with real space, is no longer considered a part of pure mathematics; like mechanics and physics, it belongs among the applications of mathematics.

**HERMANN WEYL, 1885 TO 1955**

The ultimate truths of mathematics, then, cannot be established by any experimental proof that the deductions from them are true; since the supposed experimental proof takes them for granted.

**HERBERT SPENCER, 1885 – 1977**

It is a platitude that pure mathematics can have unexpected consequences and affect even daily life.

**J.E. LITTLEWOOD, 1885 – 1977**

Pure mathematics exist by themselves; no will produces them, no power can limit them. They are eternal laws that no man can infringe, and from which it is impossible to escape.

**S. SANDARAM IYER, 1883**

- Saturn has a moon called **Mimas** and it has a crater in it that makes it look like the **Death Star**.
- When the **red supergiant Betelgeuse** explodes and becomes a **supernova**, which could happen any time between today and a million years from now, it will be visible from Earth, even during the day, and will shine brighter than a full moon for months before fading.
- In 2022, you'll be able to see two stars collide. The stars are too dim to see with the naked eye, but astronomers have been watching them slowly spiral closer and closer together. Their collision will cause a giant explosion (**AKA a Red Nova**), which will then fizzle out into a dimmer, but permanently-visible addition to the night sky.

The invention of logarithms came to the world as a bolt from the blue. No previous work had led up to it... It stands isolated, breaking in upon human thought abruptly, without borrowing from the work of other intellects or following known lines of mathematical thought.

**JOHN MOULTON, 1844 – 1921**

Division is esteemed one of the busiest operations of Arithmetic, and such as requireth a mind not wandering, or settled upon other matters.

**THOMAS HYLLES, THE ARTE OF VULGAR ARITHMETICKE, 1600**

The literary convention that numbers less than 10 should be given in words is often highly unsuitable in mathematics... The excessive use of the word forms is regrettably spreading at the present time.

**J.E. LITTLEWOOD, 1885 – 1977**

What exactly is mathematics? Many have tried but nobody has really succeeded in defining mathematics; it is always something else. Roughly speaking, people know that it deals with numbers, figures, with relations, operations, and that its formal procedures involving axioms, proofs, lemmas, theorems have not changed since the time of Archimedes.

**STAN ULAM, 1909 – 1984**

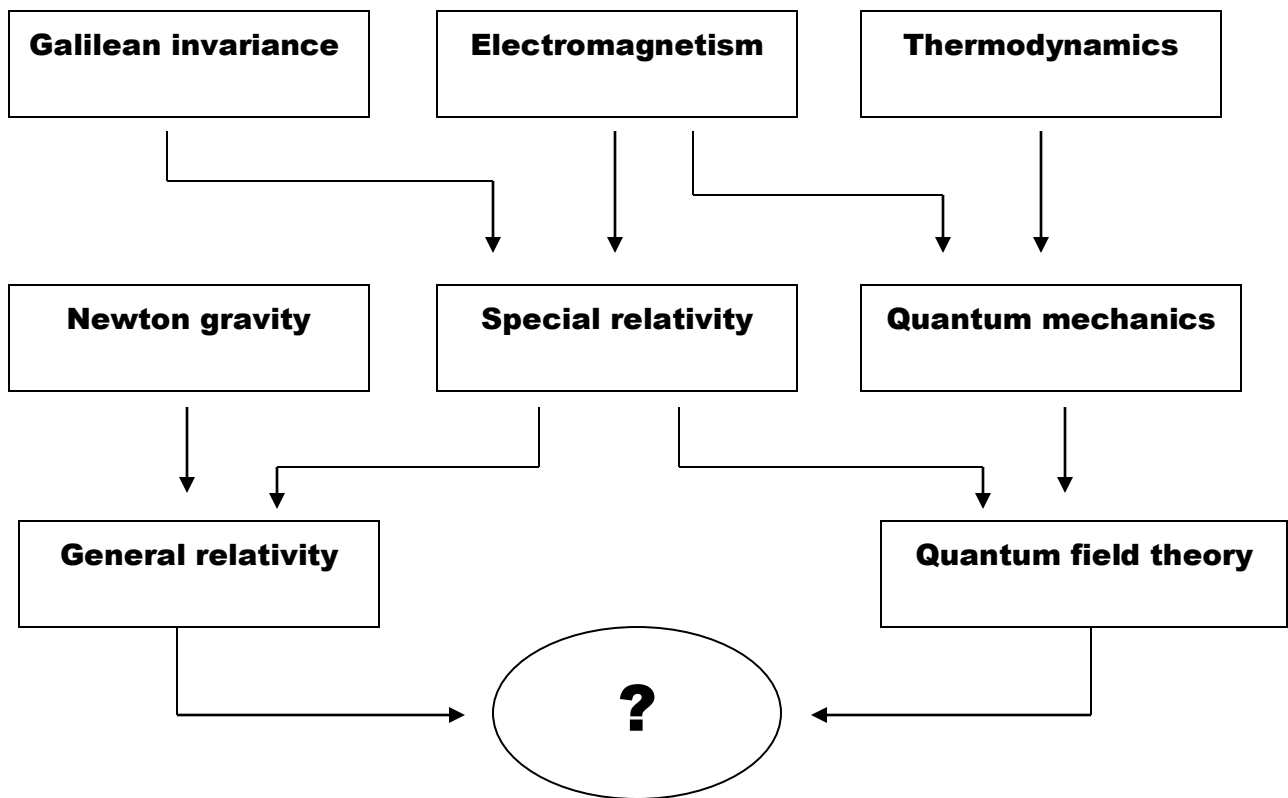
- The **Dragonfly 44 galaxy** is as big as the Milky Way, and is 99.99% dark matter.
- In order for **astronauts** to live for a year in space they have to drink 730 liters of their own recycled sweat and urine because water is extremely limited.

**Earth has a powerful magnetic field – this phenomenon is caused by nickel-iron core of the planet**

If you were to compress the entire Earth down to the size of a **peanut**, you would have a black hole.

|   |   |
|---|---|
| <b>special relativity + quantum mechanics</b> | Relativistic quantum electrodynamics<br>( <b>very precise and highly successful</b> ) |
| <b>quantum mechanics + gravity</b>            | Theories of quantum gravity<br>( <b>no data to test them</b> )                        |

**Electroweak force splits into electromagnetic and weak forces (around  $10^{-6}$  seconds after the big bang)**





**NASA has discovered a "water world" planet 40 light-years away that may contain exotic materials such as "hot ice" and "superfluid water".**



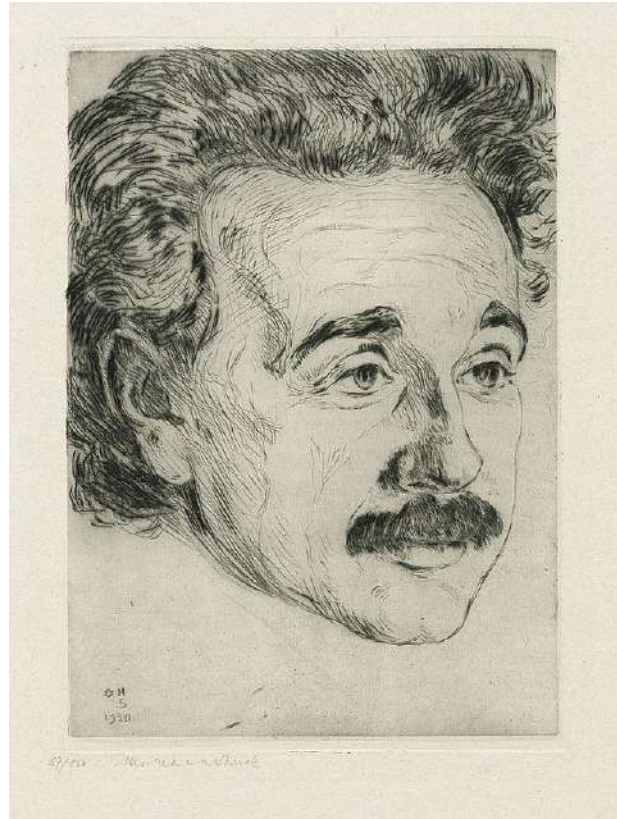
| Theory of relativity  | Quantum mechanics  |
|---|--|
| <ul style="list-style-type: none"> <li>• Removes inconsistencies in the classical theory.</li> <li>• Describes the behavior of matter at high energies and high speeds</li> </ul> | <ul style="list-style-type: none"> <li>• Removes disagreements between theory and experiments</li> <li>• Describes the behavior of microscopic particles.</li> </ul> |

**At small distances large 'quantum fluctuations' in the gravitational field make gravity very strongly-interacting.**

**The effort to understand the universe is one of the very few things that lifts human life a little above the level of farce, and gives it some of the grace of tragedy.**

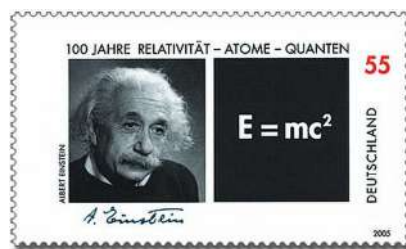



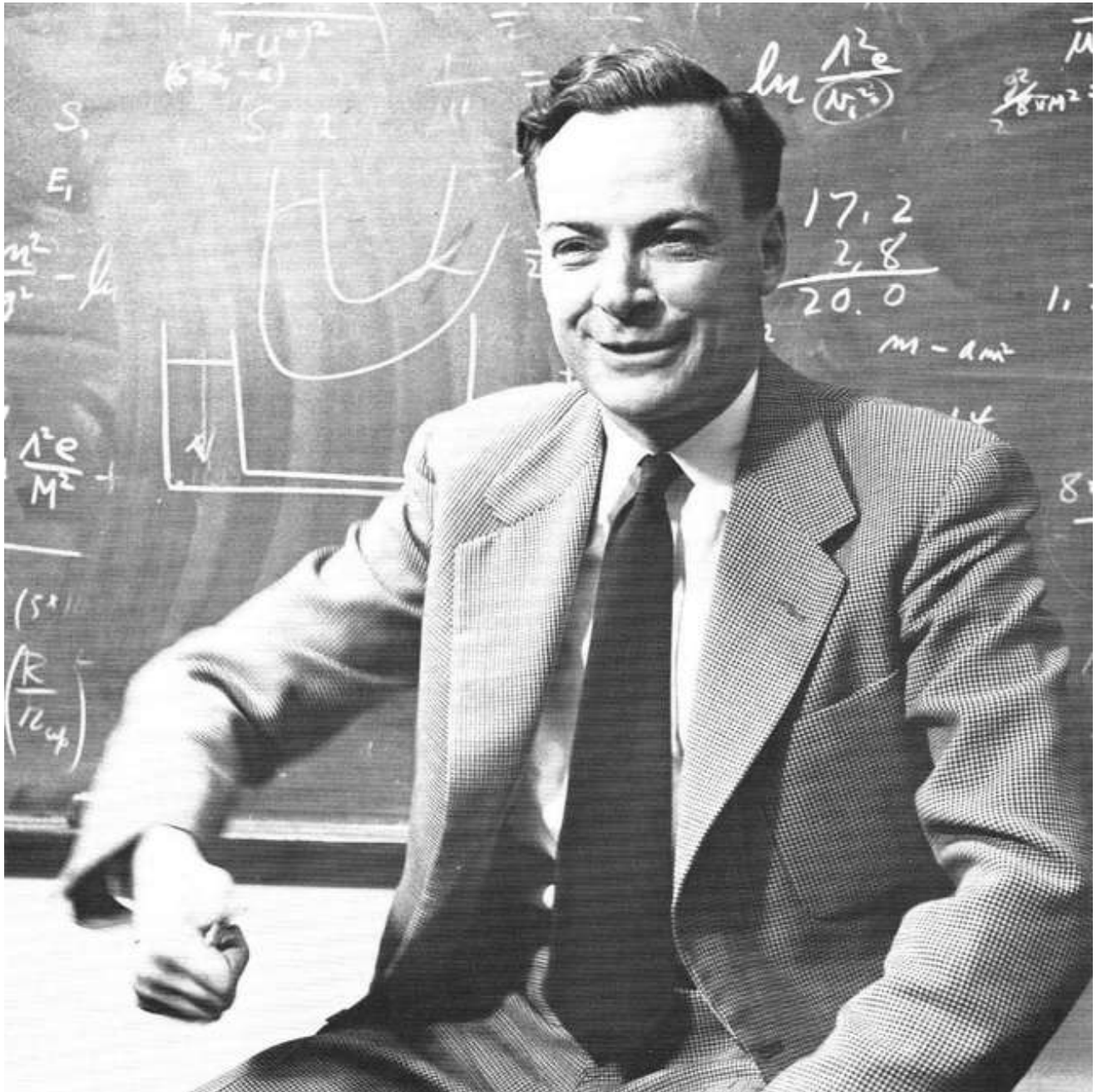
**Steven Weinberg**



**Black holes do not suck in objects. They have an extremely strong gravitational force which is almost impossible to overcome by any object that comes too close to it. If our Sun was replaced with a black hole of equal mass, the planets would orbit around the black hole as they orbit the Sun today.**

According to the NASA, the speed of Earth rotation is gradually slowing and it's happening at a rate of 1.4 milliseconds per 100 years. You may think it's not a big deal. But if you add up that small discrepancy every day for years and years, it can make a very big difference indeed. At this speed, the day may become 25 hours after 140 million years.

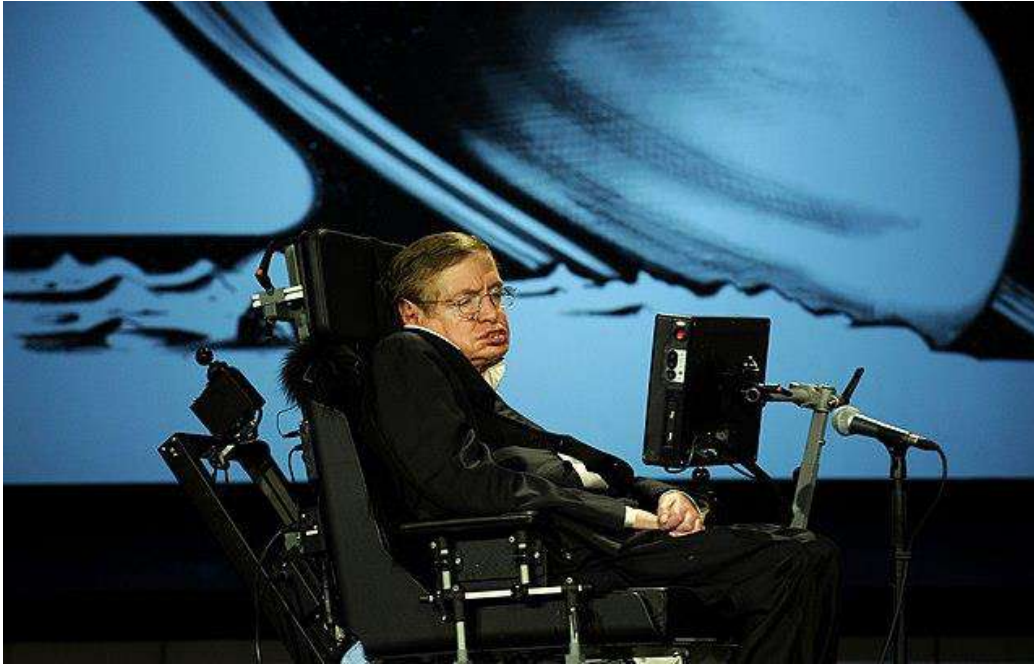




I... a universe of atoms, an atom in the universe.

- Richard Feynman

## 25 Inspiring Stephen Hawking quotes that question our Universe



**Dr. Stephen Hawking**, a professor of mathematics at the University of Cambridge, delivers a speech entitled "Why we should go into space" during a lecture that is part of a series honoring NASA's 50th Anniversary, Monday, April 21, 2008, at George Washington University's Morton Auditorium in Washington

Doctors predicted Stephen Hawking would not live past the age of 23 when he was 21. He slightly outlived the odds, living an additional 55 years. Hawking, a brilliant astrophysicist, passed away at the age of 76, leaving behind a sizable body of work that has contributed to our understanding of the cosmos. Here is a collection of the best Stephen Hawking quotations that capture some of his insight and wisdom for the rest of us. Hawking's theories on black holes, space-time, and the Big Bang may take more than a basic interest in physics to understand.

1. "We are just an advanced breed of monkeys on a minor planet of a very average star. But we can understand the Universe. That makes us something very special."
2. "If time travel is possible, where are the tourists from the future?"

3. "Ever since the dawn of civilization, people have not been content to see events as unconnected and inexplicable. They have craved an understanding of the underlying order in the world. Today we still yearn to know why we are here and where we came from. Humanity's deepest desire for knowledge is justification enough for our continuing quest. And our goal is nothing less than a complete description of the universe we live in."
4. "If aliens ever visit us, I think the outcome would be much as when Christopher Columbus first landed in America, which didn't turn out very well for the Native Americans."
5. "One of the basic rules of the universe is that nothing is perfect. Perfection simply doesn't exist...Without imperfection, neither you nor I would exist."
6. "I don't think the human race will survive the next thousand years, unless we spread into space. There are too many accidents that can befall life on a single planet. But I'm an optimist. We will reach out to the stars."
7. "We each exist for but a short time, and in that time explore but a small part of the whole universe."
8. "There could be whole antiworlds and antipeople made out of antiparticles. However, if you meet your antiself, don't shake hands! You would both vanish in a great flash of light."
9. "The human race is just a chemical scum on a moderate-sized planet, orbiting around a very average star in the outer suburb of one among a hundred billion galaxies. We are so insignificant that I can't believe the whole universe exists for our benefit. That would be like saying that you would disappear if I closed my eyes."

10. "If aliens ever visit us, I think the outcome would be much as when Christopher Columbus first landed in America, which didn't turn out very well for the Native Americans."
11. "Today will still yearn to know why we are here and where we came from. Humanity's deepest desire for knowledge is justification enough for our continuing quest. And our goal is nothing less than a complete description of the universe we live in."
12. "The most remarkable property of the universe is that it has spawned creatures able to ask questions."
13. "...only in the few universes that are like ours would intelligent beings develop and ask the question: "Why is the universe the way we see it?" The answer is then simple: If it had been any different, we would not be here! "
14. The eventual goal of science is to provide a single theory that describes the whole universe.
15. "The message of this lecture is that black holes ain't as black as they are painted. They are not the eternal prisons they were once though...things can get out of a black hole both on the outside and possibly to another universe. So if you feel you are in a black hole, don't give up – there's a way out."
16. "We now know that our galaxy is only one of some hundred thousand million that can be seen using modern telescopes, each galaxy itself containing some hundred thousand million stars."
17. "We have developed from the geocentric cosmologies of Ptolemy and his forebears, through the heliocentric cosmology of Copernicus and Galileo, to the modern picture in which the earth is a medium-sized planet orbiting around an average star in the outer suburbs of an ordinary spiral galaxy, which is itself only one of about a million million galaxies in the observable universe."

18. "The discovery that the universe was expanding was one of the great intellectual revolutions of the twentieth century."
19. "Although quantum mechanics has been around for nearly 70 years, it is still not generally understood or appreciated, even by those that use it to do calculations."
20. "The most important point: that the universe is governed by a set of rational laws that we can discover and understand."
21. "In space no one can hear you scream; and in a black hole, no one can see you disappear."
22. "Black holes are not really black after all: they glow like a hot body, and the smaller they are, the more they glow."
23. "Even if there is only one possible unified theory, it is just a set of rules and equations. What is it that breathes fire into the equations and makes a universe for them to describe? "
24. "Another prediction of general relativity is that time should appear to run slower near a massive body like the earth."
25. "No boundary condition: The idea that the universe is finite but has no boundary (in imaginary time)."



**Splitting of atoms → Fission**

**Joining of atoms → Fusion**

### **Spontaneous Fission**



The spontaneous non-induced splitting of the unstable heavier element nuclei into two almost equal fragments (lighter element nuclei) resulting in the release of a huge amount of energy and more than one neutron on the average

**Recent results from the experiment reveal that the visible cosmos is just 4.9% composed of normal matter.**

**What characteristics does dark matter have?**

**What is this "dark energy" exactly?**

**NEW FUNDAMENTAL FORCES?**

**ADDITIONAL DIMENSIONS?**

**Cosmologists are working to find answers to these issues.**



## Conclusion:

For those who readily take on the God Hypothesis, most of the big answers remain unquestioned. But for those of us who don't believe in idol God, most of the big questions remain unanswered.

### No need for God...

Big Bang didn't need mysterious God to ignite the spark and set the world running. Our universe could have rushed into existence 13.9 billion years ago as a result of just the physical laws and constants being there. Spontaneous creation is the reason so that we could be here today to justify something rather than nothing, why it is that we and the universe go to all the bother of existing.

No one created the universe. No one guides our destiny. No heaven. No hell either. This leads us to a deep understanding that there is no God to direct our fate and reset the universe with humans in mind. We have this one short life to wonder what shaped human understanding of the universe and beyond. Today we know better, but a complete description of the cosmos remains elusive. The only thing that freaks us more than its creation is its grand purpose of creation. There is mathematics everywhere. It can be seen in the things we make and the artistic creations we value. Even though we might not realise it, mathematics can be found everywhere around us in the surroundings, wildlife and plant varieties, and even in human beings. Humans developed math as a tool to precisely describe their surroundings. It's true that in the life of researchers and technologists, numbers are present everywhere. Even before humans developed it, mathematics was present in nature. From the properties of subatomic particles and the realm of quantum physics to the fundamental workings of the universe, math proves unquestionably effective in describing and predicting their physical reality. If you reside close to gardens, farms, and forests, you might search for a downed tree to count the rings on or search for a structure created by a spider (cobweb), which is made up of nearly flawless concentric circles. Our galaxy is shaped like a Fibonacci spiral. The paths on which the planets circle the sun are concentric. Additionally, we notice concentric circles in Saturn's rings. However, a question that lies at the intersection of philosophy and science arises: **Is Math the Language of the Universe?**

## Best Cosmology **Books of All Time:**



- **A Brief History of Time**  
Book by Stephen Hawking
- **Cosmos**  
Book by Carl Sagan
- **The Fabric of the Cosmos**  
Book by Brian Greene
- **The Elegant Universe**  
Book by Brian Greene
- **The Big Picture: On the Origins of Life, Meaning, and the Universe Itself**  
Book by Sean Carroll
- **A Universe from Nothing: Why There Is Something Rather Than Nothing?**  
Book by Lawrence M. Krauss
- **The First Three Minutes: A Modern View of the Origin of the Universe**  
Book by Steven Weinberg
- **A Short History of Nearly Everything**

Book by Bill Bryson

- **The Grand Design**

Book by Stephen Hawking

- **Death by Black Hole: And Other Cosmic Quandaries**

Book by Neil deGrasse Tyson

- **Pale Blue Dot: A Vision of the Human Future in Space**

Book by Carl Sagan

- **The Hidden Reality: Parallel Universes and the Deep Laws of the Cosmos**

Book by Brian Greene

- **The Universe in a Nutshell**

Book by Stephen Hawking

- **Astrophysics for People in a Hurry**

Book by Neil deGrasse Tyson

- **Parallel Worlds: A Journey through Creation, Higher Dimensions, and the Future of the Cosmos**

Book by Michio Kaku

- **Big Bang: The Origin of the Universe**

Book by Simon Singh

- **Our Mathematical Universe: My Quest for the Ultimate Nature of Reality**

Book by Max Tegmark

- **A Briefer History of Time**

Book by Stephen Hawking, Leonard Mlodinow

- **Reality Is Not What It Seems: The Journey to Quantum Gravity**

Book by Carlo Rovelli, Simon Carnell, Erica Segre

- **Einstein: His Life and Universe**

Book by Walter Isaacson

- **Black Holes and Time Warps: Einstein's Outrageous Legacy**

Book by Kip S. Thorne, Stephen Hawking

- **The Demon-haunted World: Science As a Candle in the Dark**

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- **Origins: Fourteen Billion Years of Cosmic Evolution**

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- **Brief Answers to the Big Questions**

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- **Black Holes and Baby Universes**

Book by Stephen Hawking

- **The Black Hole War: My Battle with Stephen Hawking to Make the World Safe for Quantum Mechanics**

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- **Welcome to the Universe: An Astrophysical Tour**

Book by Neil deGrasse Tyson

- **The Holographic Universe: The Revolutionary Theory of Reality**

Book by Michael Talbot

- **The Road to Reality: A Complete Guide to the Laws of the Universe**

Book by Roger Penrose

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- **The Goldilocks Enigma: Why Is the Universe Just Right for Life?**

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- **The Whole Shebang: A State-of-the-Universe[s] Report**

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- **The Cosmic Landscape: String Theory and the Illusion of Intelligent Design**

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- **Einstein's Dreams**

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- **Time Reborn: From the Crisis in Physics to the Future of the Universe**

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- **The Trouble with Physics: The Rise of String Theory, the Fall of a Science and What Comes Next**

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Book by Jorge Cham, Daniel Whiteson

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- **The Greatest Story Ever Told—So Far: Why Are We Here?**

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- **The Universe in Your Hand: A Journey Through Space, Time, and Beyond**

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- **The Case for a Creator: A Journalist Investigates Scientific Evidence That Points Toward God**

Book by Lee Strobel

- **Astrophysics and the Evolution of the Universe (Second Edition)**

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- **Wonders of the Universe**

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Book by Fritjof Capra

- **Cosmicomics**

Book by Italo Calvino, William Weaver

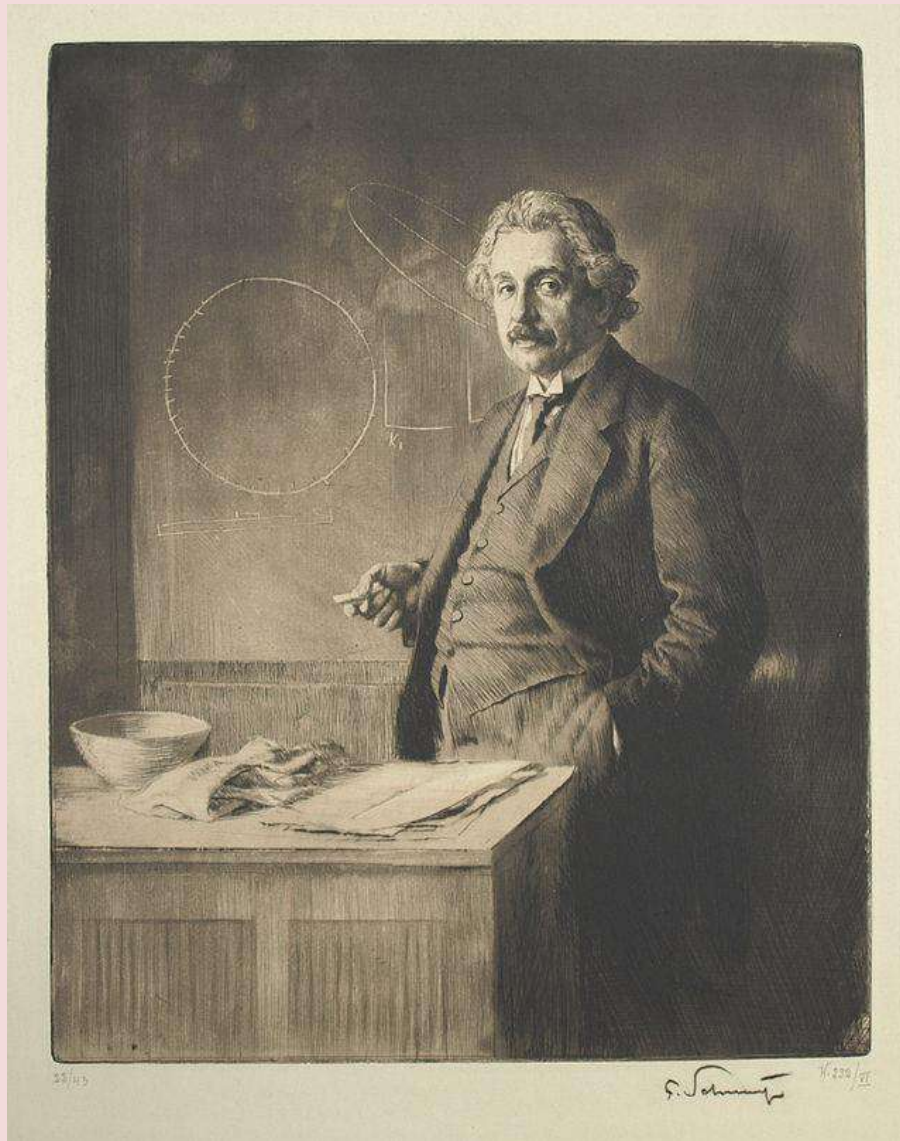


## **One final thought:**

If you feel that this information has been useful to you, please take a moment to share it with your friends on LinkedIn, Facebook and Twitter. Think about leaving a quick review on **Google Play Books** if you think this book has given you insight into the grand narrative of the cosmos from a fresh, inspired perspective and you have learnt something valuable.

**Cosmology** is a study area that combines the astronomy and physics in an endeavour to comprehend the physical universe as a cohesive whole. It is both incredibly fun and fascinating. I want to spread my passion to as many individuals as I can. I also hope that this isn't the end of your quest for solutions to the mysteries that have plagued mankind since its beginning. What role does humanity have in the universe's 14 billion year history? What role does humanity play in the history of this planet? How does humanity participate in the complex chain of life here on Earth?

**Thank you!**



"The important thing is not to stop questioning. Curiosity has its own reason for existence. One cannot help but be in awe when he contemplates the mysteries of eternity, of life, of the marvelous structure of reality. It is enough if one tries merely to comprehend a little of this mystery each day."

— **Albert Einstein**



**Albert Einstein as a child**



**Stephen Hawking** being presented by his daughter **Lucy Hawking** at the lecture he gave for NASA's 50th anniversary



The area formula for the entropy—or number of internal states—of a black hole suggests that information about what falls into a black hole may be stored like that on a record, and played back as the black hole evaporates.

"I believe the simplest explanation is, there is no God. No one created the universe and no one directs our fate. This leads me to a profound realization that there probably is no heaven and no afterlife either. We have this one life to appreciate the grand design of the universe and for that, I am extremely grateful."

— **Stephen Hawking**



WE BECOME MORE AWARE OF HOW LITTLE WE KNOW ABOUT HOW THE COSMOS FUNCTIONS AS WE LEARN MORE ABOUT IT. THE UNIVERSE IS FULL OF SECRETS, FROM THE DOORSTEP OF OUR OWN SOLAR SYSTEM TO THE FAR-OFF SHORES OF THE INTERGALACTIC OCEAN. WITH THE HELP OF A LARGE NUMBER OF TELESCOPES AND SATELLITES, WE HAVE INCREASED OUR UNDERSTANDING OF THE UNIVERSE. WE HAVE BEEN INVESTIGATING THE HISTORY OF THE COSMOS, FROM THE BIG BANG THROUGH COMETS' PECULIARITIES AND OUR CURIOSITY ABOUT THE CHEMISTRY OF STARS. ONE THING UNITES MANY OF THE MOST PREVALENT THEORIES: THEY START FROM A MATHEMATICAL FRAMEWORK THAT AIMS TO EXPLAIN MORE THAN OUR EXISTING LEADING THEORIES CAN. WE HUMANS, INQUISITIVE CREATURES SHAPED BY DARWIN'S THEORY OF NATURAL SELECTION, ARE USED TO ASKING QUESTIONS. THE QUESTION IS NOT 'DO WE KNOW EVERYTHING FROM THE VERY NATURE OF PHYSICAL LAWS TO THE UNDERLYING DISCOMFORT OF THE ULTIMATE QUESTION OF OUR PLACE IN THE UNIVERSE?' OR IT IS 'DO WE KNOW ENOUGH?' BUT HOW DOES THE CREATIVE PRINCIPLE RESIDE IN MATHEMATICS? THERE'S SOMETHING VERY MATHEMATICAL ABOUT OUR GIGANTIC COSMOS, AND THAT THE MORE CAREFULLY WE LOOK, THE MORE EQUATIONS ARE BUILT INTO NATURE. FROM BASIC ARITHMETIC TO THE CALCULATION OF ROCKET TRAJECTORIES, MATH PROVIDES A GOOD UNDERSTANDING OF THE EQUATIONS THAT GOVERN THE WORLD AROUND US. OUR UNIVERSE ISN'T JUST DESCRIBED BY MATH, BUT THAT UNIVERSE IS A "GRAND BOOK" WRITTEN IN THE LANGUAGE OF MATHEMATICS. WE FIND IT VERY APPROPRIATE THAT MATHEMATICS HAS PLAYED A STRIKING ROLE IN OUR EXPANDING UNDERSTANDING OF THE UNIVERSE – ITS ORIGIN, COMPOSITION AND DESTINY.



IS THE UNIVERSE  
MADE OF MATH?

## MATHEMATICAL UNIVERSE

OUR SEARCH FOR THE ULTIMATE NATURE OF REALITY



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